7th International Conference on

New Developments In Photodetection

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NDIP

Introduction to particle and radiation interactions with matter

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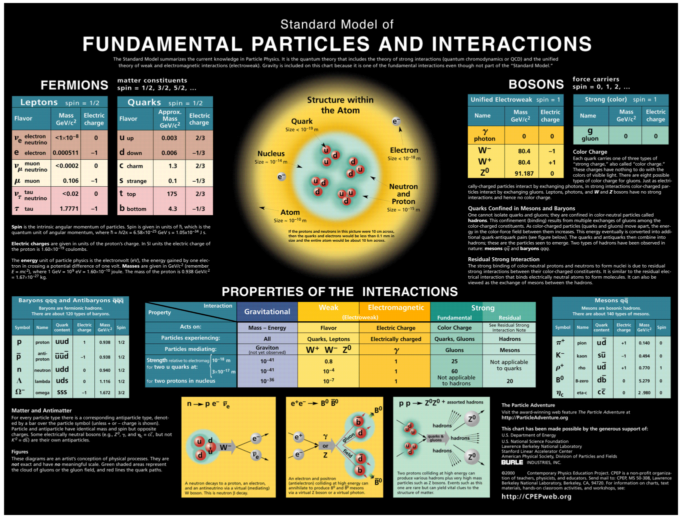
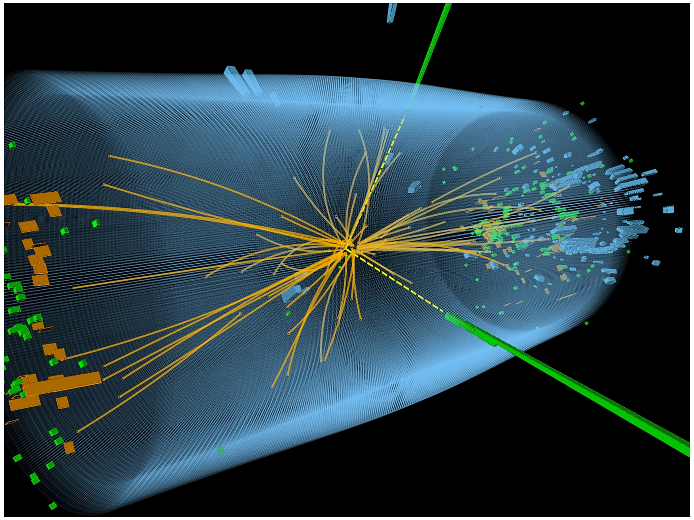




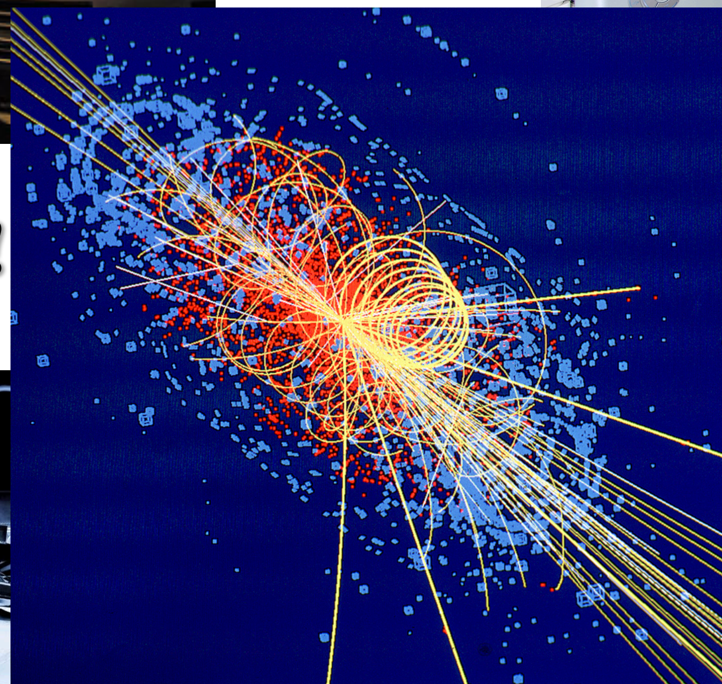
Plan:

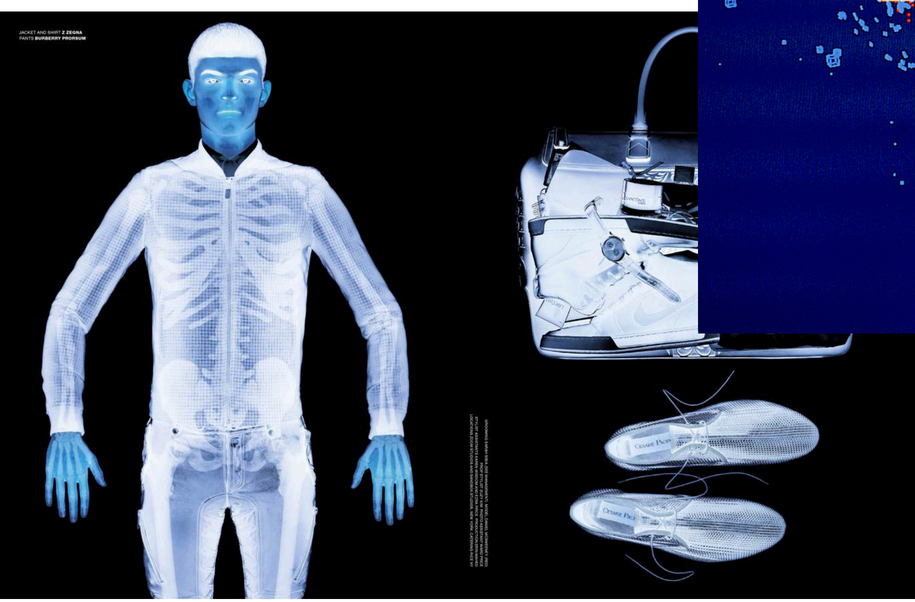
1. Introduction
2. Energy loss of charged particles in matter
3. Interactions of Photons
4. Some examples for light detection
5. Summary

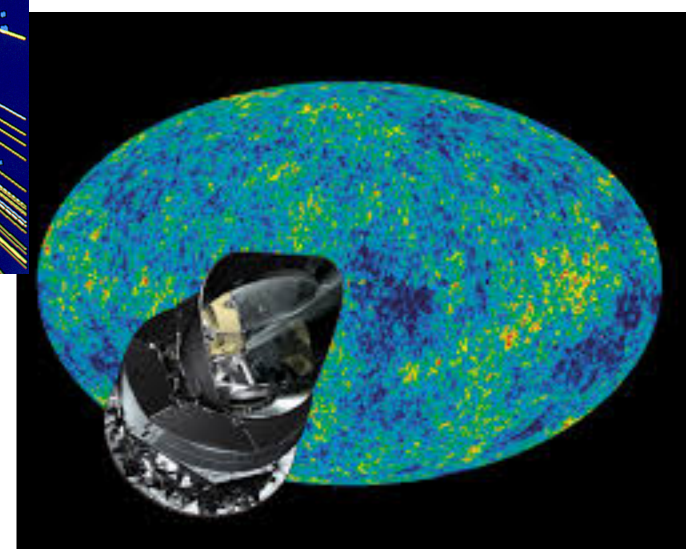
1. Introduction

Why should we care?

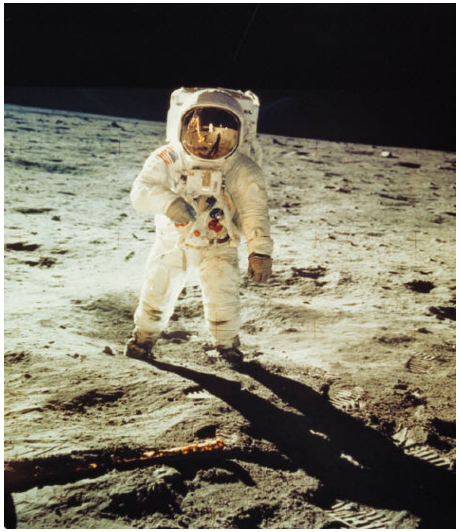
Affects all of our life!







We are radioactive !



On average a human of 70 kg has 17 mg of Potassium 40

This results in 4,4 kBq of activity
This is 4400 disintegrations per second !

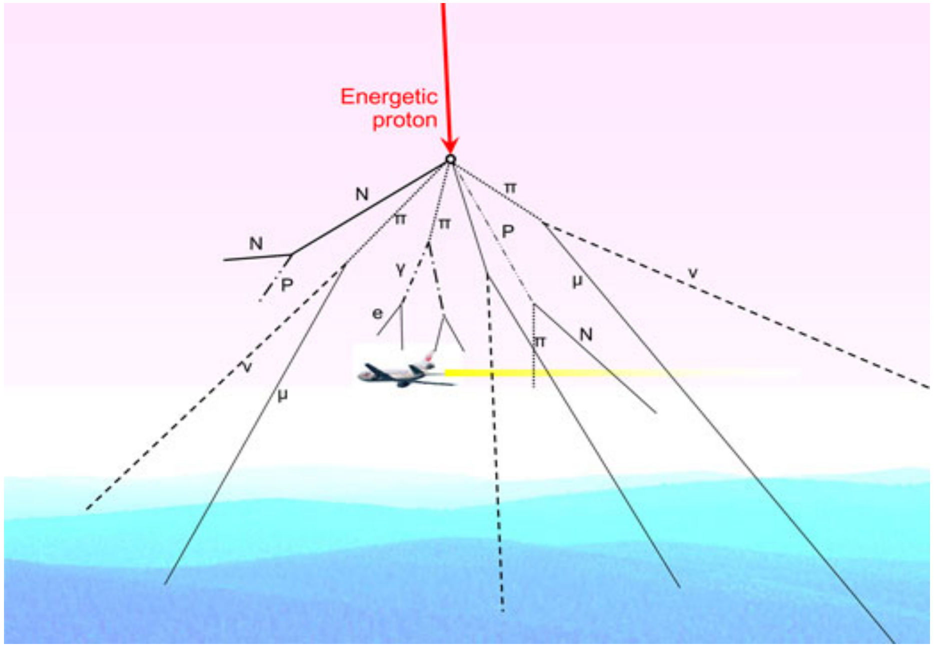
$$\beta^- : {}^{40}_{19}K \rightarrow {}^{40}_{20}Ca + e^- + \overline{\nu}_e$$

$$EC : {}^{40}_{19}K + e^- \rightarrow {}^{40}_{18}Ar^* + \nu_e \rightarrow {}^{40}_{18}Ar^* + \nu_e \rightarrow {}^{40}_{18}Ar + \gamma + \nu_e$$

$$\beta^+ : {}^{40}_{19}K \rightarrow {}^{40}_{18}Ca + e^+ + \nu_e$$

And when you eat a healthy carrot you get 0,1 kBq / kg !

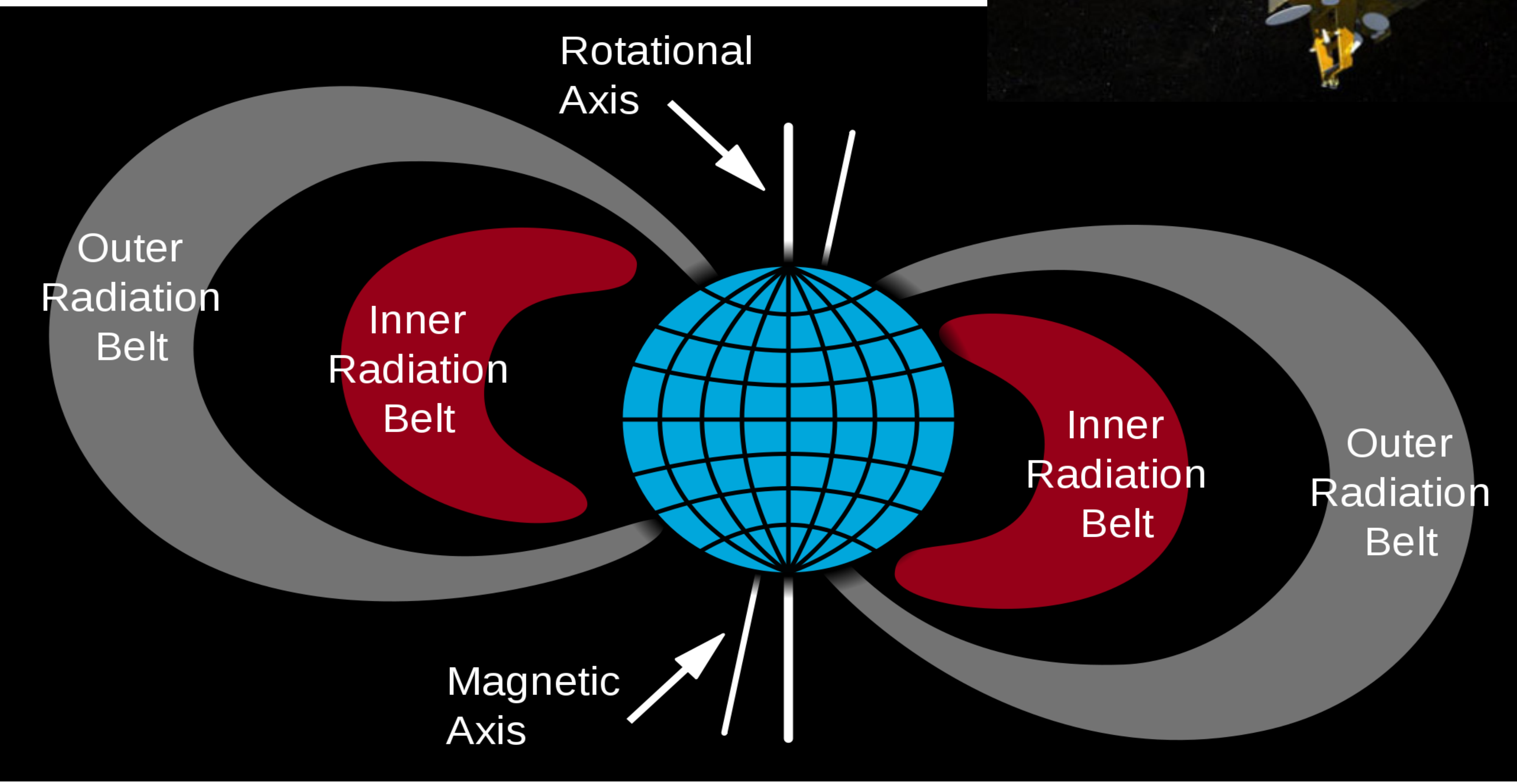
Exposure in flight:



- The lowest dose rate measured was 3 μSv h^{-1} during a Paris-Buenos Aires flight.
- The highest rates were 6.6 μSv h^{-1} during a Paris to Tokyo flight and 9.7 μSv h^{-1} on the Concorde in 1996–1997.
- The corresponding annual effective dose, based on 700 hours of flight for subsonic aircraft and 300 hours for the Concorde, can be estimated at between 2 mSv for the least exposed routes and 5 mSv for the more exposed routes.

- Sv = Unit of equivalent dose
- Natural annual background: (0.4 – 4) mSv
- Limit for radiation workers = 20 mSv y^{-1}

High level electronics in Satellites is affected



Two basic reasons why to detect particles or radiation:

1. **Study** the interactions of particles and radiation to understand the basic laws of nature

Applied and fundamental research

2. **Use** fundamental interactions to study the characteristics of matter

Basic quantities:

$$\hbar c = 197,326960\text{Mev fm}$$

Fine structure constant: $$\alpha = \frac{e^2}{4\pi\epsilon_0}\frac{1}{\hbar c} = 1/137,03599976$$

Classical electron radius: $$r_e = \frac{e^2}{4\pi\epsilon_0}\frac{1}{m_e c^2} = \alpha\frac{\hbar c}{m_e c^2} = 2,817940285 \times 10^{-15}\text{m}$$

Energy loss: $$K = 4\pi N_A r_e^2 m_e c^2 = 4C m_e c^2 = 0,307\text{MeV.cm}^2.\text{g}^{-1}$$

More on orders of magnitude:

Basic units used in particle physics to describe detectors:

- **Photon absorption coefficient μ : $I = I_0 e^{-\mu x}$**
- **Radiation length X_0 : $E = E_0 e^{-x/X0}$**
- **Nuclear interaction length λ_I : $e^{-x/\lambda I}$**

Material	X_0 (g/cm^2) (cm)		λ_I (g/cm^2) (cm)	
H	61.28	(866)	50.8	(715.5)
C	42.7	(18.8)	86.3	(38.1)
Scintillator	43.7	(42.4)	81.9	(79.3)
Fe	13.84	(1.76)	131.9	(16.7)
Xe	8.48	(2.87)	169.	(29.1)
Pb	6.37	(0.56)	194.	(17.1

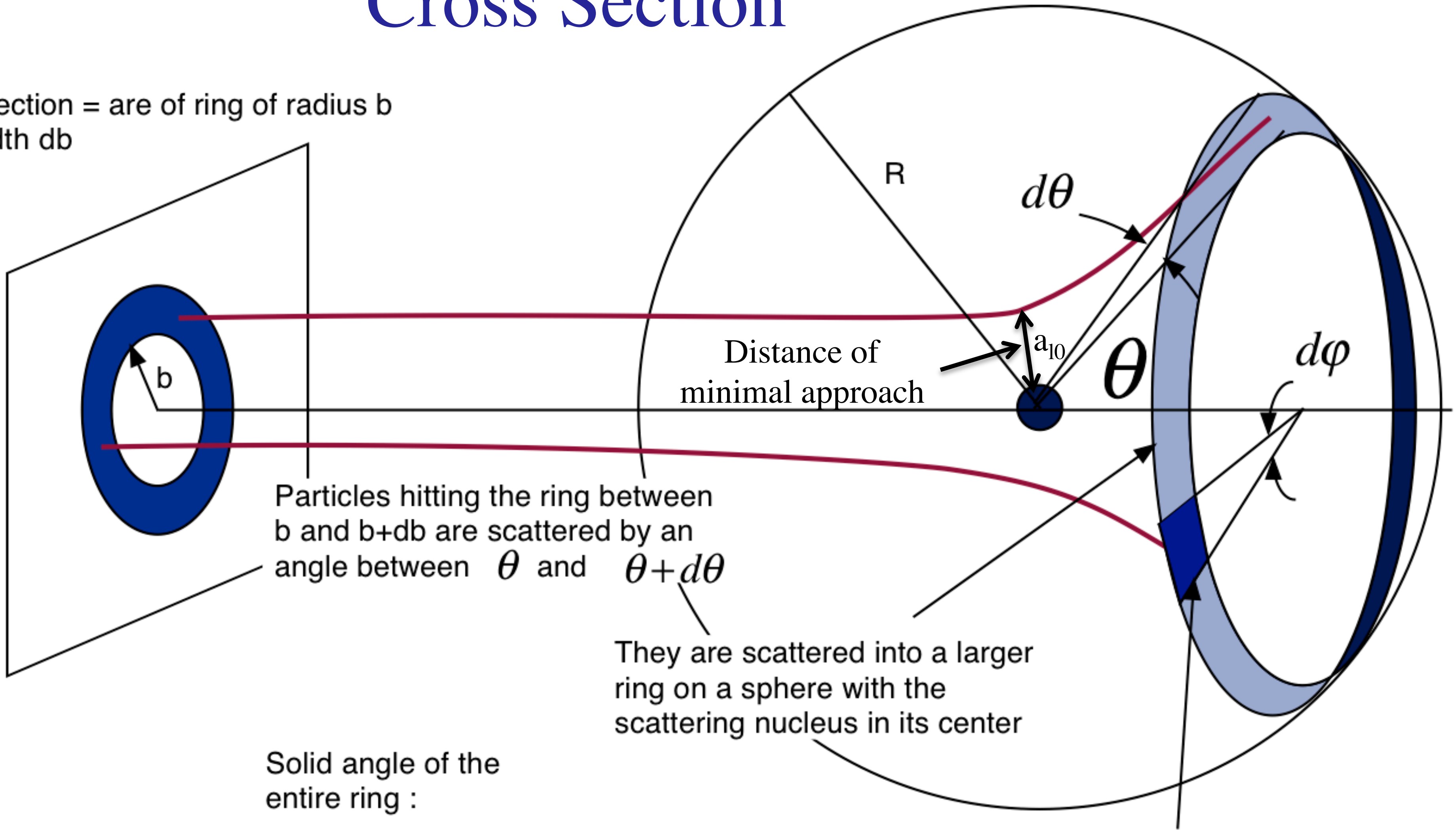
Related cross sections:

Strong interaction : σ ~ 10 ÷ 100 mb
Electro-magnetic interaction: σ ~ 10 ÷ 100 nb
Weak interaction: σ ~ 10 ÷ 100 pb

(1 barn = 10^{-28} m^2)

Cross Section

cross section = are of ring of radius b
and width db



Solid angle of the
entire ring :

$$d\Omega = \frac{2\pi R\sin(\theta)Rd\theta}{R^2} = 2\pi\sin(\theta)d\theta$$

solid angle of
small area:

$$d\Omega = \frac{d\varphi R\sin(\theta)Rd\theta}{R^2} = \sin(\theta)d\theta d\varphi$$

$$P(\Theta) = N \times \frac{\Delta S}{R^2} \times \frac{b}{\sin\Theta} \times \left|\frac{db}{d\Theta}\right|$$

Number of scatters
[atoms/cm^2]

Solid angle
dΩ

Differential cross section
$\sigma(\Theta)$ [cm^2]

Differential cross section: $$\sigma(\Theta) = \frac{b}{\sin\Theta} \times \left|\frac{db}{d\Theta}\right|$$

Total cross section: $$\sigma_{tot} = \int_0^{4\pi} \sigma(\Theta) d\Omega$$

In practice the target is a slab of material
We want to know the average number of interactions per unit time scattered into dΩ

- Assume the target centers uniformly distributed and the slab not to thick (no secondary interactions)
- The number of centers seen by the beam = **Ndx**
- N = density of centers, **$N = N_A \times \rho/A$** (N_A = Avogadro's number, A = Atomic number)
- **dx** = thickness of the slab along the beam

average number of interactions per unit time scattered into dΩ:

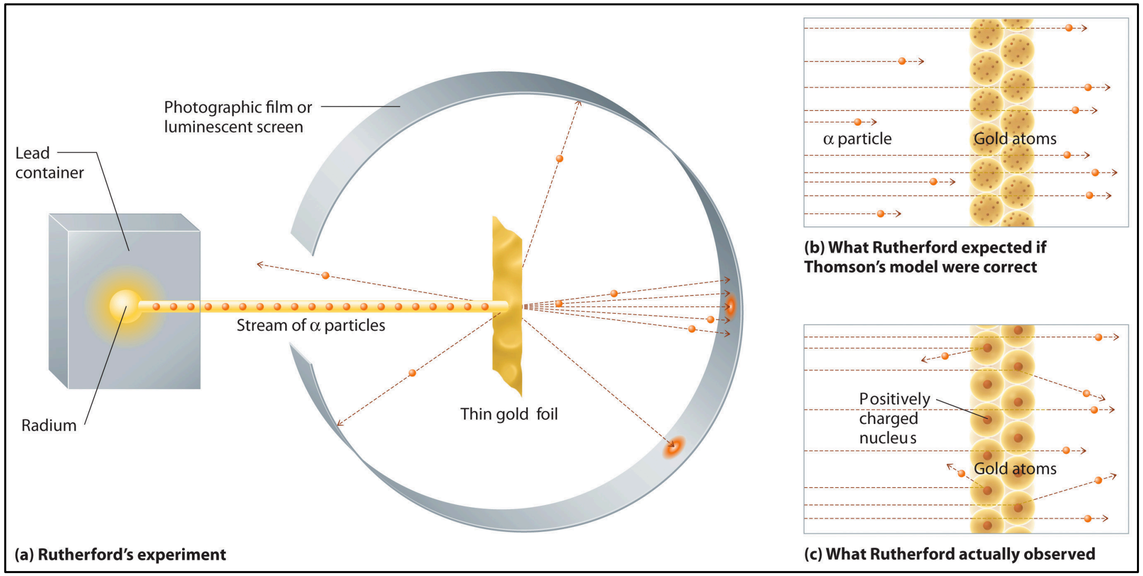
$$N_{scattered} = F \times S \times N \times dx \times \frac{d\sigma}{d\Omega}$$

F = flux per unit time [particles/(time x cm^2)]
S = target area [cm^2]

The total number of scattered into all angles is:

$$N_{tot} = F \times S \times N \times dx \times \sigma$$

Example: Coulomb Scattering (Rutherford)



$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

b = impact parameter
θ = scattering angle

Relation between scattering angle, impact parameter and shortest approach:

$$\mathrm{tg}\left(\frac{\theta}{2}\right) = \frac{a_0}{2b},\ \mathrm{a}_0 = \text{shortest distance of approach, b} = \text{impact parameter}$$

$$\mathrm{b} = \frac{a_0}{2}\cot\left(\frac{\theta}{2}\right) \quad \text{and} \quad \frac{db}{d\theta} = \frac{a_0}{4}\times\frac{1}{\sin^2\left(\frac{\theta}{2}\right)}$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \boxed{\frac{a_0^2}{16\sin^4\left(\frac{\theta}{2}\right)}} \quad \text{with } \mathrm{a}_0 = \frac{kZ_1Z_2e^2}{E},\ k = {}^{1}\!/_{4\pi\varepsilon_0},\ Z_1, Z_2 = \text{atomic numbers of beam and target}$$

Application:

We have a beam of protons with an energy of 22 MeV and an intensity of 200 nA on a thin gold foil target with a thickness e = 100 μg / cm^2.

Question: How many protons are detected in a detector with a surface S = 0.2 cm^2 at a distance, R = 10 cm and an angle θ = 10°?

$$N_{inc} = \frac{I}{q} = \frac{200 \times 10^{-9}\ \text{A}}{1.602 \times 10^{-19}\ \text{A s}} = 1.25 \times 10^{12}\ \text{particles/sec}$$

$$\text{Number of detected particles} = \text{N}_{\text{det}} = N_{inc} \times N_{target} \times \Omega \times \sigma(\theta)$$

$$N_{target} = \frac{N_{Avogardo} \times Thickness}{\text{atomic mass}} = \frac{6 \times 10^{23}(nucl/mol) \times 100 \times 10^{-6}(g/cm^2)}{197(g/mol)}$$

$$\Omega = \frac{S}{R^2} = \frac{0.2\ \text{cm}^2}{100\ \text{cm}^2}$$

$$\sigma(\theta) = \frac{a_0^2}{16\sin^4\left(\frac{\theta}{2}\right)} = \frac{1}{16\sin^4\left(\frac{\theta}{2}\right)} \times \left[\frac{kZ_1Z_2e^2}{E}\right]^2 \quad \text{using} \quad \frac{\text{ke}^2}{\hbar\text{c}} = 1/137 \text{ and } \hbar\text{c} = 200 \text{ MeV fm}$$

$$\text{N}_{\text{det}} = 1.25 \times 10^{12}\ \text{particles/sec} \times \frac{6 \times 10^{23}(nucl/mol) \times 100 \times 10^{-6}(g/cm^2)}{197(g/mol)} \times \frac{0.2\ \text{cm}^2}{100\ \text{cm}^2} \times \left[\frac{\text{ke}^2}{\hbar\text{c}} \times \hbar\text{c} \times \frac{1 \times 79}{22\ \text{MeV}}\right]^2 \times \frac{1}{16\sin^4\left(\frac{\theta}{2}\right)}$$

$$N_{\text{det}} = 1.25 \times 10^{12} \frac{protons}{\text{sec}} \times \frac{3 \times 10^{17}}{cm^2} \times 2 \times 10^{-3} \left[\frac{1}{137} \times 200\ \times 10^{-13}\ \text{MeV cm}^2 \times \frac{1 \times 79}{22\ \text{MeV}}\right]^2 \times 10^3$$

$$N_{\text{det}} = 2 \times 10^5 \text{protons/sec.}$$

2. Energy loss of charged particles in matter

Energy loss of particles (1):

Charged Particles:

Light particles: electrons & positrons

- Bremsstrahlung dominates @ E> 20 MeV
- Inelastic scattering with atoms (ionization)
- Elastic scattering with nucleons
- Cherenkov Radiation
- Nuclear reactions

Heavy particles: muons, protons, π, α

- Inelastic scattering with atoms (ionization): $\sigma \approx 10^{-17} - 10^{-16}$ cm^2
- Elastic scattering with nucleons
- Cherenkov Radiation
- Nuclear reactions
- Bremsstrahlung

Energy loss of particles (2):

Neutral particles

photons

- Photoelectric Effect
- Compton Scattering
- Pair Production
- Nuclear reactions (small contribution)

neutrons

Slowing down (moderation) by:

- Elastic Scattering
- Inelastic Scattering

-> Nuclear Absorption
-> Nuclear Reactions (fission)

neutrinos

Electro-weak interaction:

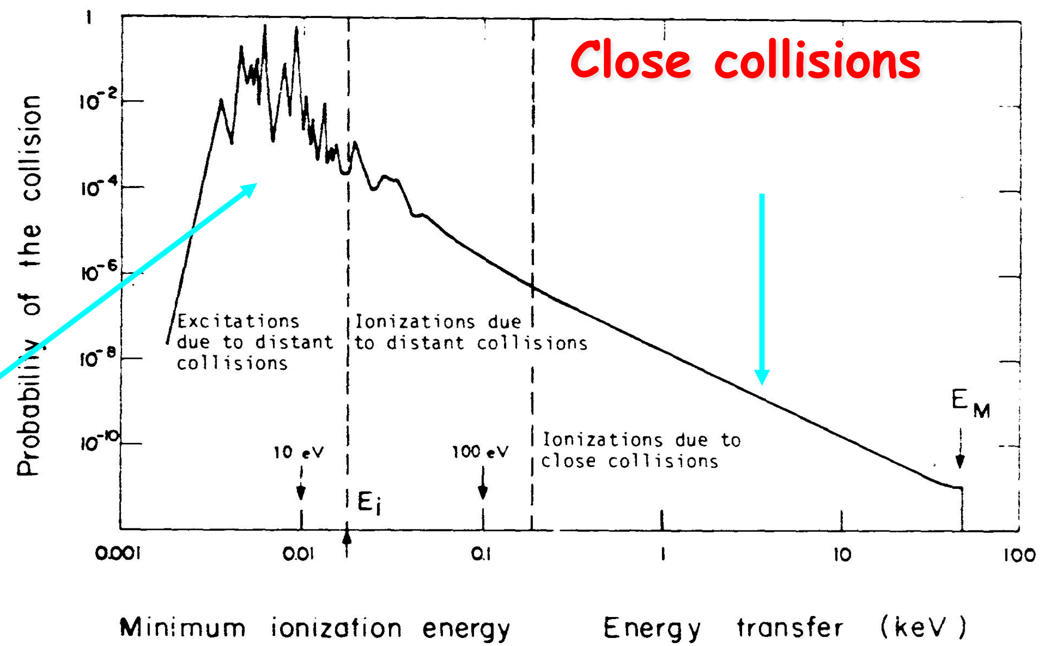
$\nu_l + e^- \rightarrow e^- + \nu_l$

$\nu_l + N \rightarrow l + X$

Energy loss of heavy particles by ionization

A heavy particle, M, looses its energy in matter in a continues way by transferring it on electrons.

Dependent on the distance of the interaction, the energy loss is more or less important.



Distant collisions

Maximum energy transfer:

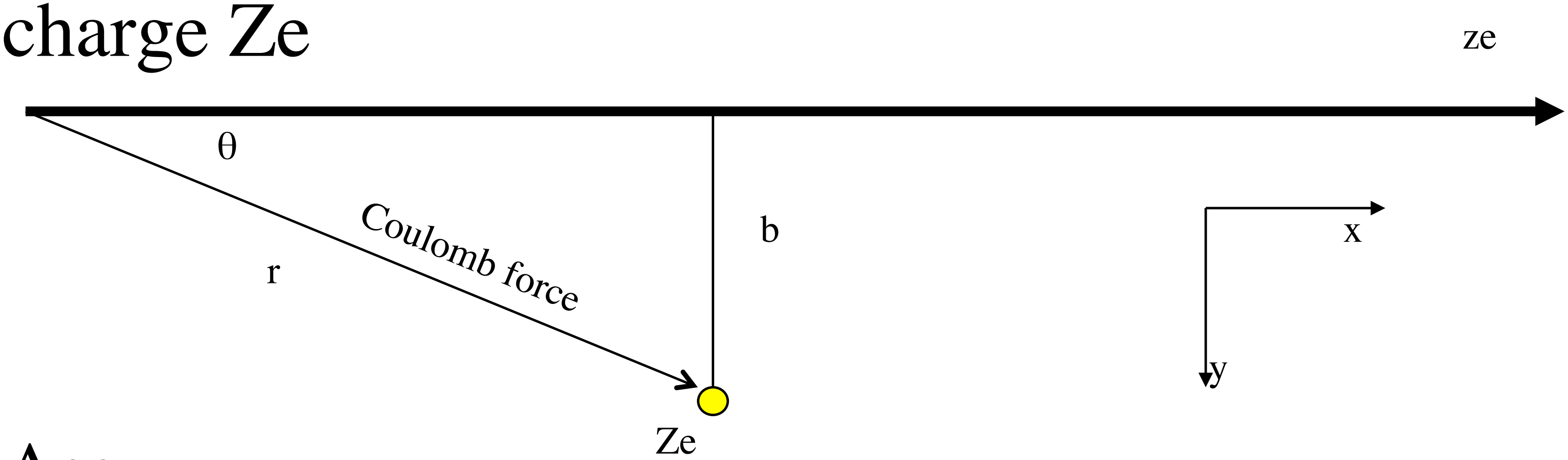
$$T_{max} = \frac{2\gamma^2 M^2 m_e v^2}{m_e^2 + M^2 + 2\gamma m_e M}$$

Bethe-Bloch Formula

- Describes how heavy particles ($m >> m_e$) loose energy when travelling through material
- Exact theoretical treatment difficult
  - Atomic excitations
  - Screening
  - Bulk effects

Bethe-Bloch (1)

- Consider particle of charge ze, passing a stationary charge Ze



- Assume
  - Target is non-relativistic
  - Target does not move
- Calculate
  - Energy transferred to target

Force on projectile

$$\vec{F} = \frac{z \times Z \times e^2}{4\pi\varepsilon_0 \times r^2} \times \frac{\vec{r}}{r}$$

$$F_b = \frac{z \times Z \times e^2}{4\pi\varepsilon_0 \times r^2} \times \frac{b}{r}$$

Change of momentum of target/projectile

$$\Delta p_b = \int_{-\infty}^{\infty} F_b dt = \int_{-\infty}^{\infty} \frac{z \times Z \times e^2}{4\pi\varepsilon_0 \times r^2} \times \frac{b}{r} dt \quad \text{using dt} = \frac{\text{dt}}{\text{dx}} dx = \frac{1}{v} dx = \frac{1}{\beta c}$$

$$\Delta p_b = \frac{z \times Z \times e^2}{4\pi\varepsilon_0 \times \beta c} \times \int_{-\infty}^{\infty} \frac{b}{r^3} dx = \frac{z \times Z \times e^2}{4\pi\varepsilon_0 \times \beta c} \times \int_{-\infty}^{\infty} \frac{b}{(\sqrt{x^2 + b^2})^3} dx$$

using Mathematica one finds:

$$\int_{-\infty}^{\infty} \frac{b}{(\sqrt{x^2 + b^2})^3} dx = \frac{2\sqrt{\frac{1}{b^2}}b}{\sqrt{b^2}} = \frac{2}{b}$$

$$\Delta p_b = \frac{z \times Z \times e^2}{2\pi\varepsilon_0 \times \beta c \times b}$$

Energy transferred

$$\Delta E = \frac{\Delta p^2}{2M} = \frac{Z^2 z^2 e^4}{2M(2\pi\varepsilon_0)^2(\beta c)^2} \frac{1}{b^2}$$

Bethe-Bloch (3)

- Consider α-particle scattering off Atom
  - Mass of nucleus: $M=A*m_p$
  - Mass of electron: $M=m_e$
- But energy transfer is

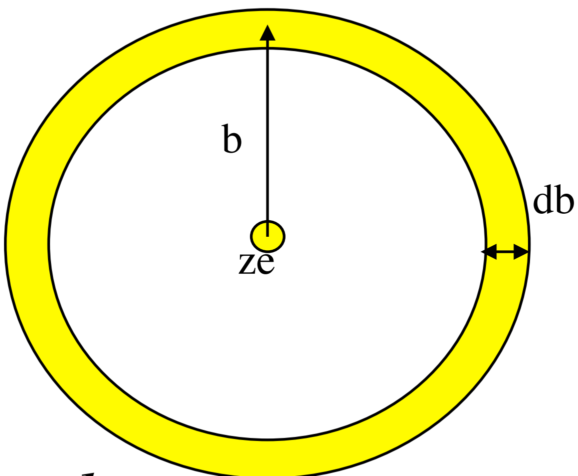
$$\Delta E = \frac{\Delta p^2}{2M} = \frac{Z^2 z^2 e^4}{2M(2\pi\varepsilon_0)^2(\beta c)^2}\frac{1}{b^2} \propto \frac{Z^2}{M}$$

- Energy transfer to single electron is

$$E_e(b) = \Delta E = \frac{2z^2 e^4}{m_e c^2 (4\pi\varepsilon_0)^2 \beta^2}\frac{1}{b^2}$$

Bethe-Bloch (4)

- Energy transfer is determined by impact parameter b
- Integration over all impact parameters



$$\frac{dn}{db} = 2\pi b \times (\text{number of electrons / unit area })$$

$$= 2\pi b \times Z \frac{N_A}{A} \rho \Delta x$$

Bethe-Bloch (5)

- Calculate average energy loss

$$\overline{\Delta E} = \int_{b_{\min}}^{b_{\max}} \mathrm{d}b \, \frac{\mathrm{d}n}{\mathrm{d}b} E_e(b) = 2C \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \Delta x \left[\ln b \right]_{b_{\min}}^{b_{\max}}$$

$$= C \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \Delta x \left[\ln E \right]_{E_{\min}}^{E_{\max}}$$

$$\text{with } C = 2\pi N_A \left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2} \right)$$

- There must be limit for E_{min} and E_{max}
  - All the physics and material dependence is in the calculation of this quantities

Bethe-Bloch (6)

- Simple approximations for
  - From relativistic kinematics

$$E_{\max} = \frac{2\gamma^2\beta^2 m_e c^2}{1 + 2\gamma \frac{m_e}{M} + \left(\frac{m_e}{M}\right)^2} \approx 2\gamma^2\beta^2 m_e c^2$$

  - Inelastic collision

$$E_{\min} = I_0 \equiv \text{ average ionisation energy}$$

- Results in the following expression

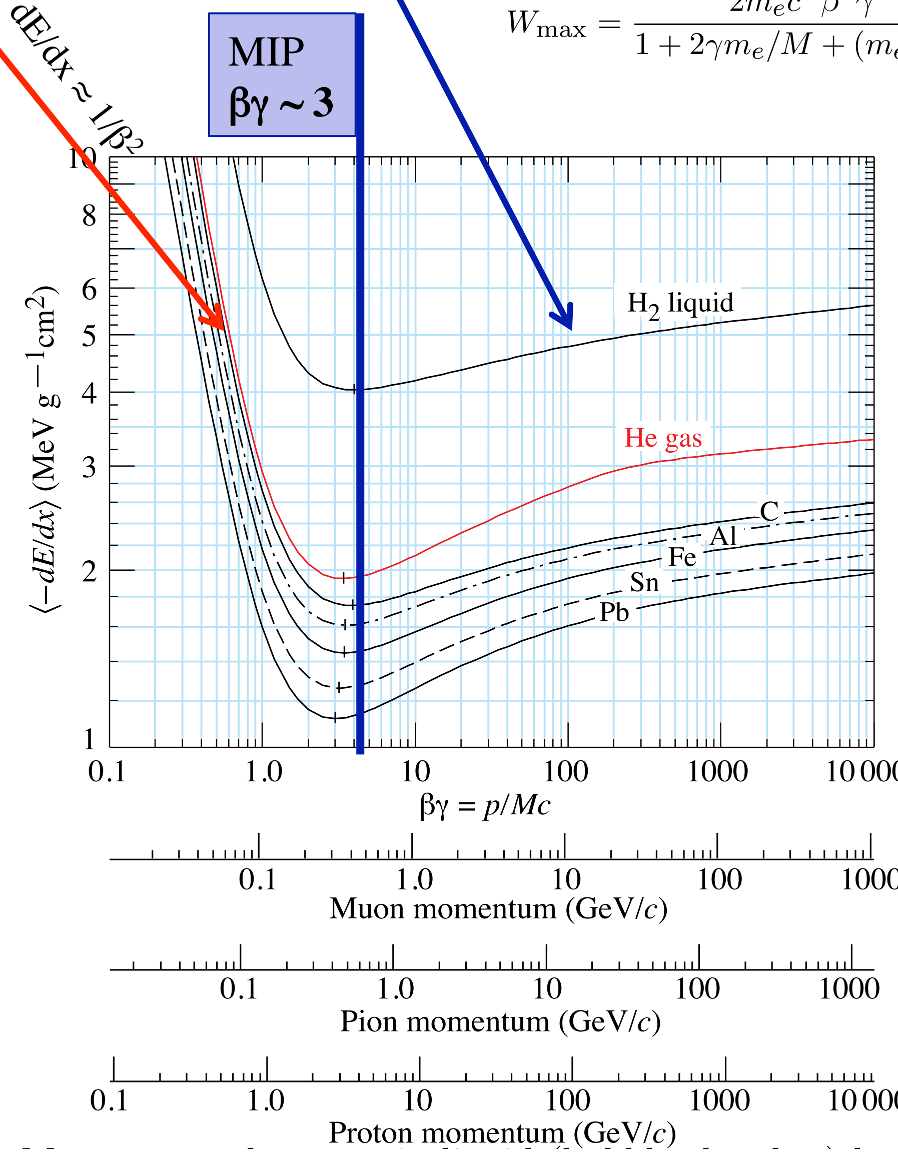
$$\frac{\overline{\Delta E}}{\Delta x} \approx 2C \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \ln\left(\frac{2\gamma^2\beta^2 m_e c^2}{I_0}\right)$$

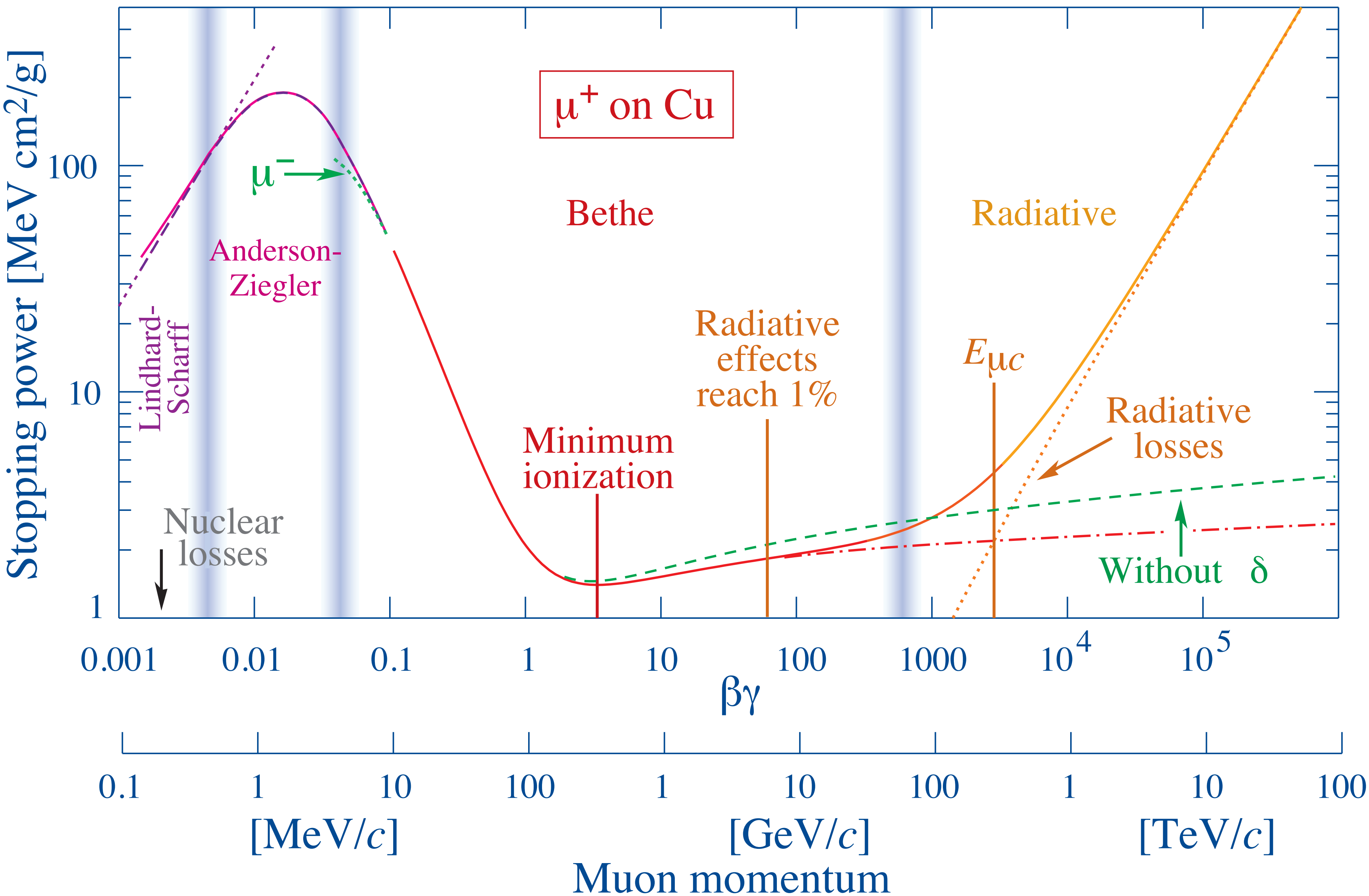
Mean Energy Loss: (Bethe-Bloch)

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

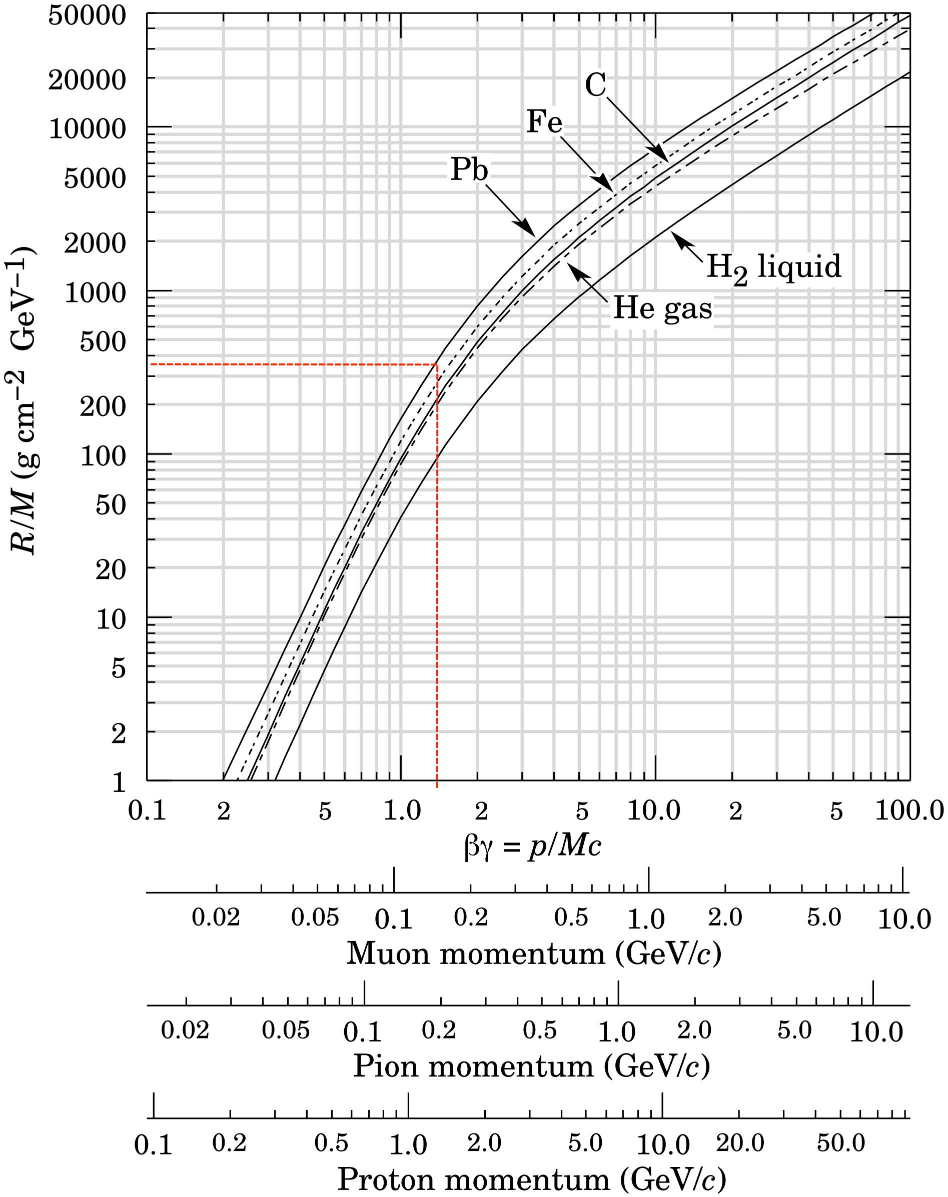
$$W_{\max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

Symbol	Definition	Value or (usual) units
α	fine structure constant $e^2/4\pi\epsilon_0\hbar c$	$1/137.035\,999\,074(44)$
M	incident particle mass	MeV/c^2
E	incident part. energy $\gamma M c^2$	MeV
T	kinetic energy, $(\gamma - 1)Mc^2$	MeV
W	energy transfer to an electron in a single collision	MeV
k	bremsstrahlung photon energy	MeV
$m_e c^2$	electron mass $\times\, c^2$	$0.510\,998\,928(11)$ MeV
r_e	classical electron radius $e^2/4\pi\epsilon_0 m_e c^2$	$2.817\,940\,3267(27)$ fm
N_A	Avogadro's number	$6.022\,141\,29(27) \times 10^{23}$ mol^{-1}
z	charge number of incident particle	
Z	atomic number of absorber	
A	atomic mass of absorber	g mol^{-1}
K	$4\pi N_A r_e^2 m_e c^2$	$0.307\,075$ MeV mol^{-1} cm^2
I	mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	density effect correction to ionization energy loss	
$\hbar\omega_p$	plasma energy $\sqrt{4\pi N_e r_e^3}\; m_e c^2/\alpha$	$\sqrt{\rho\,\langle Z/A\rangle} \times 28.816$ eV ⟶ ρ in g cm^{-3}
N_e	electron density	(units of r_e)$^{-3}$
w_j	weight fraction of the jth element in a compound or mixture	
n_j	$\propto$ number of jth kind of atoms in a compound or mixture	
X_0	radiation length	g cm^{-2}
E_c	critical energy for electrons	MeV
$E_{\mu c}$	critical energy for muons	GeV
E_s	scale energy $\sqrt{4\pi/\alpha}\; m_e c^2$	21.2052 MeV
R_M	Molière radius	g cm^{-2}




μ+ on Cu
Stopping power [MeV cm2/g]
100
10
1
μ−
Bethe
Radiative
Anderson-
Ziegler
Lindhard-
Scharff
Radiative
effects
reach 1%
Eμc
Radiative
losses
Minimum
ionization
Nuclear
losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10^4
10^5
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

Particle range:



Example: K^+ with $p_k = 700\, MeV/c$

$$m_k = 494\, MeV$$

$$\beta\gamma = \frac{p_k}{m_k c} = \frac{700}{494} = 1{,}42$$

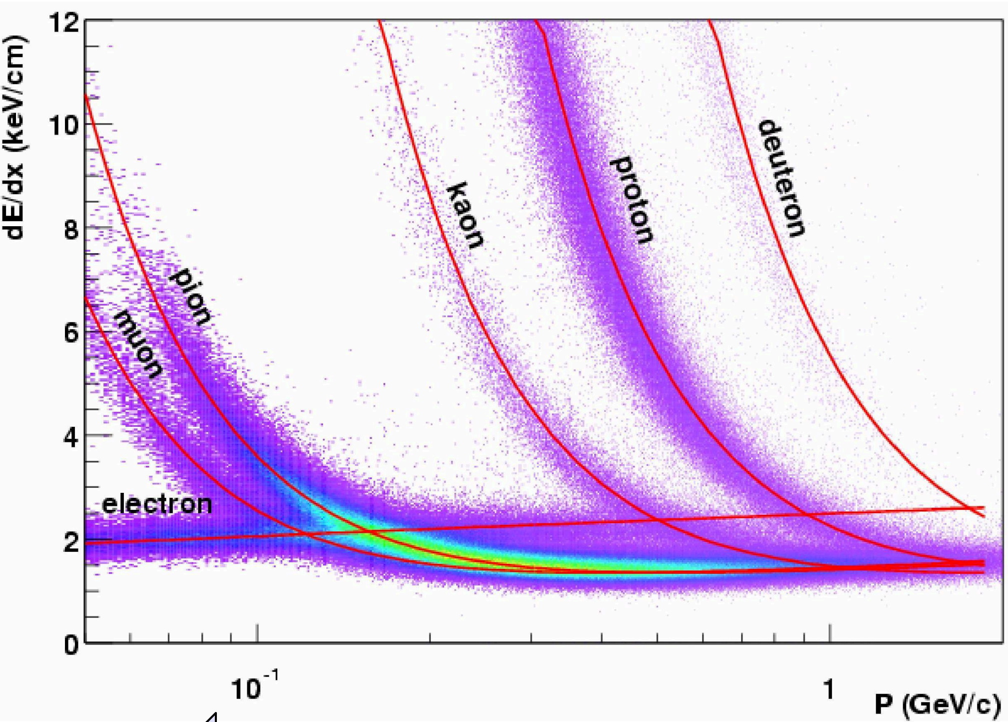
$$For Pb: R/M = 396 gcm^{-2} GeV^{-1}$$

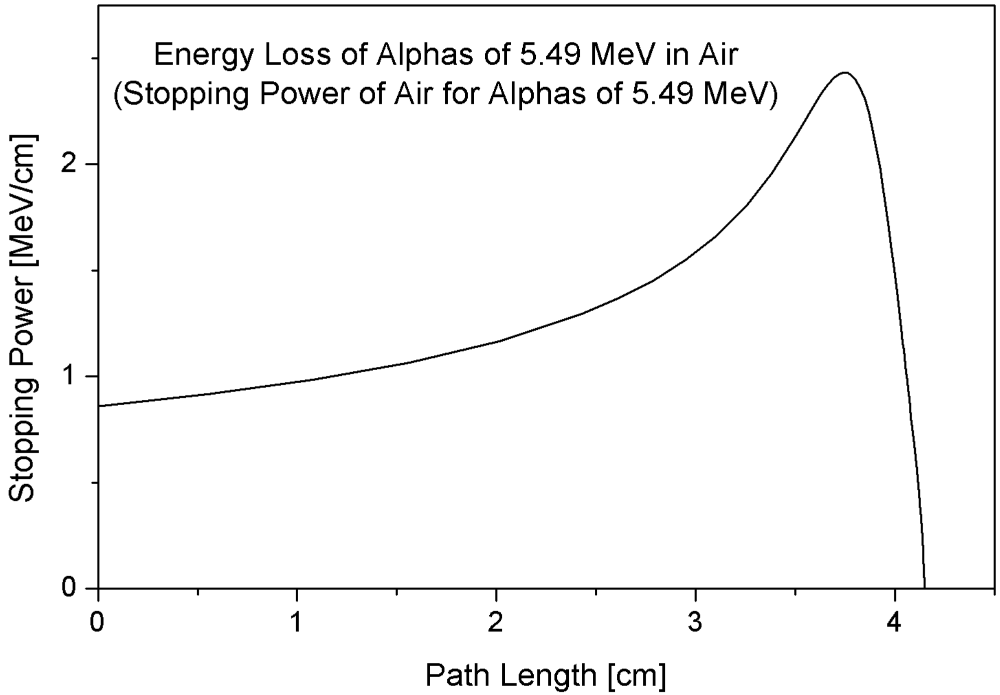
$$\Rightarrow R = 396 gcm^{-2} GeV^{-1} \times 0{,}494 GeV = 196 gcm^{-2}$$

$$\rho_{Pb} = 11{,}35 gcm^{-3}$$

$$\Rightarrow R = 196 gcm^{-2} \div 11{,}35 gcm^{-3} = \underline{\underline{17 cm}}$$

Bragg curve and Bragg peak:

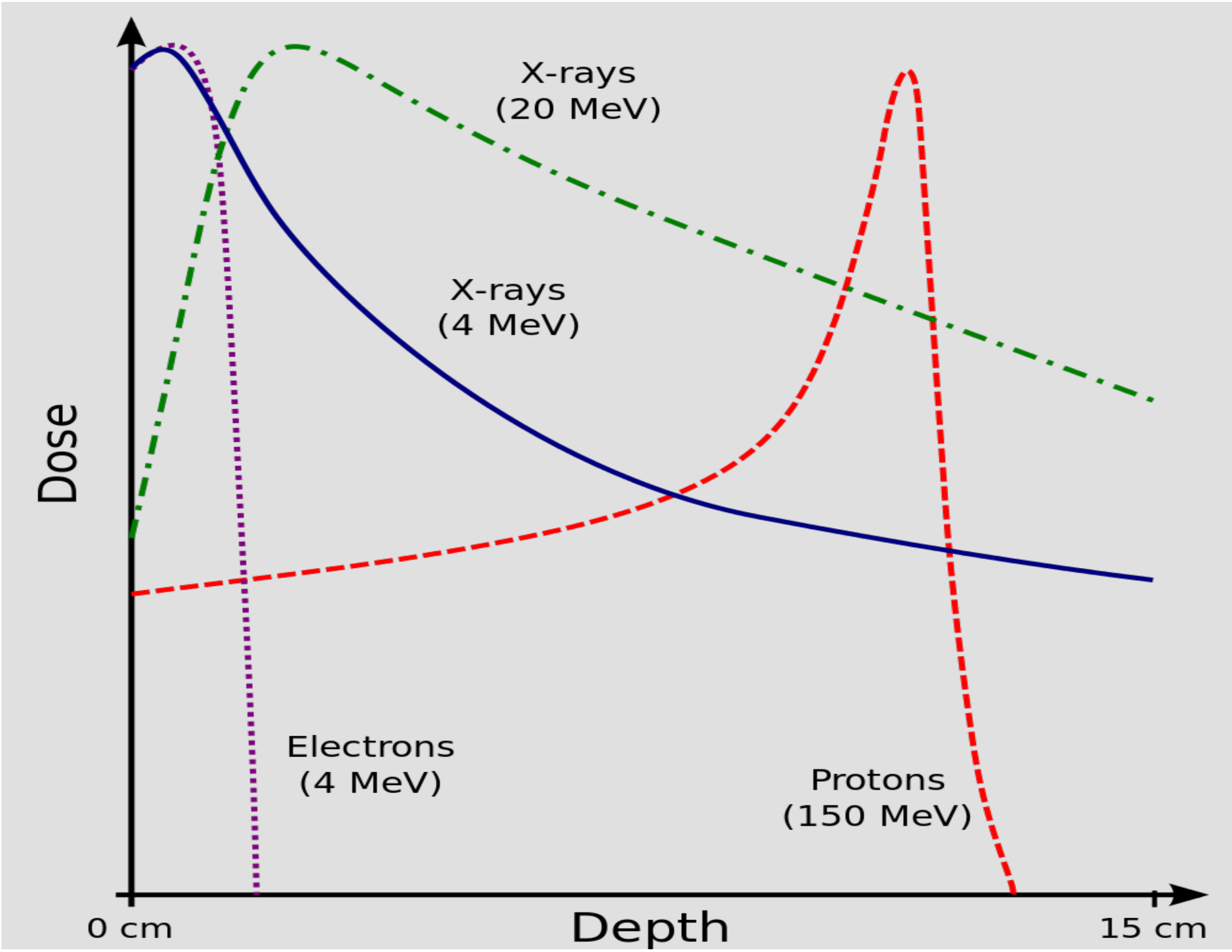




Read from right to left

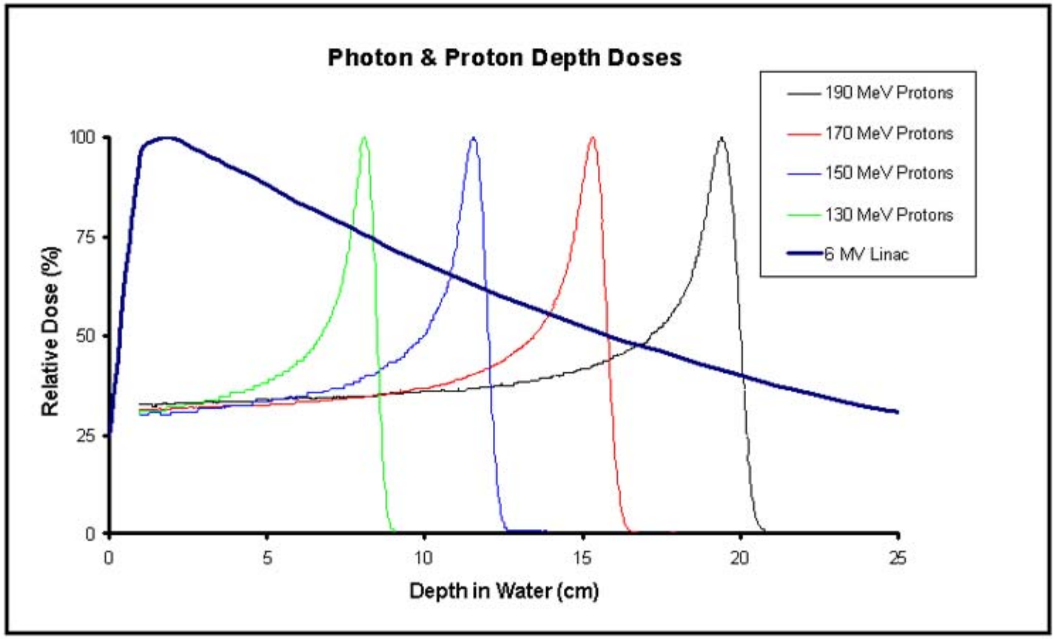
Write from left to right

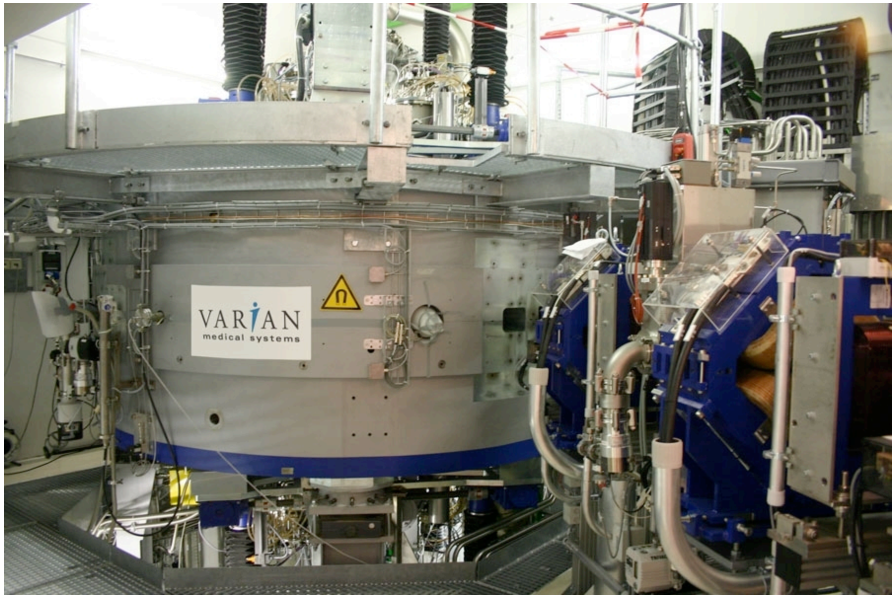
Bragg curve: Different for X-rays and heavy particles

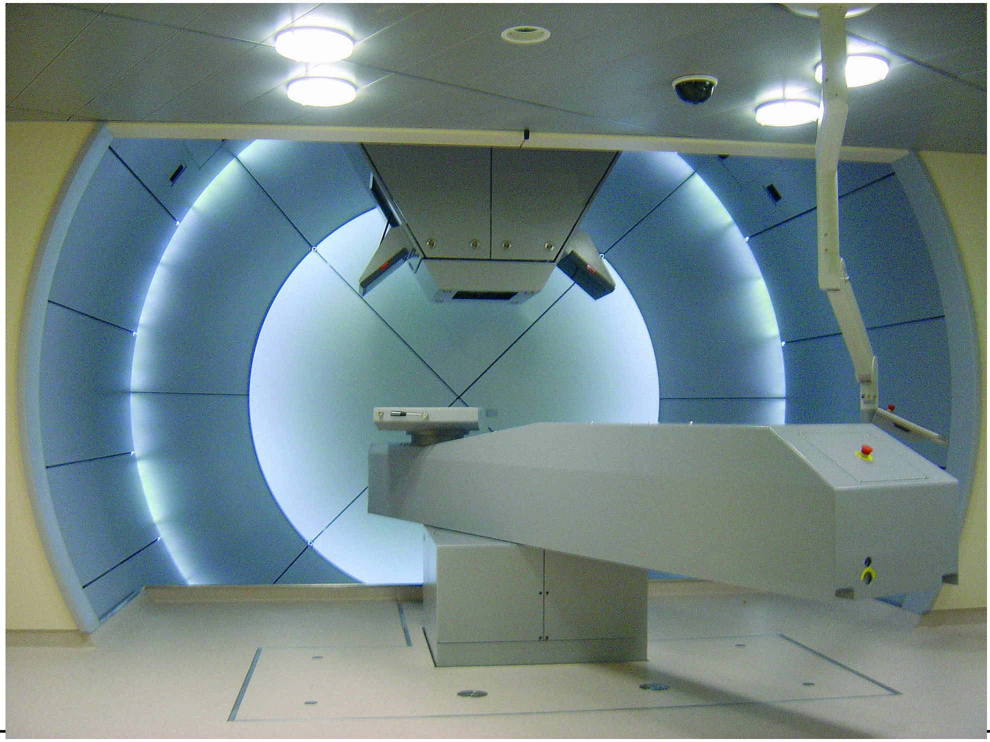


Application:

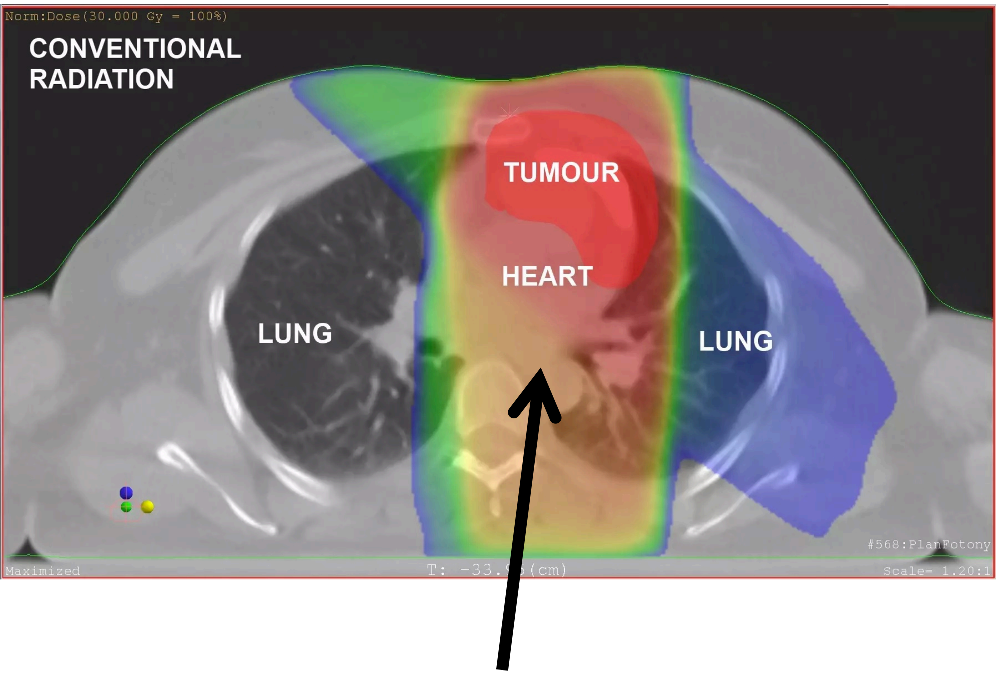
- Proton or
- Heavy Ion Therapy

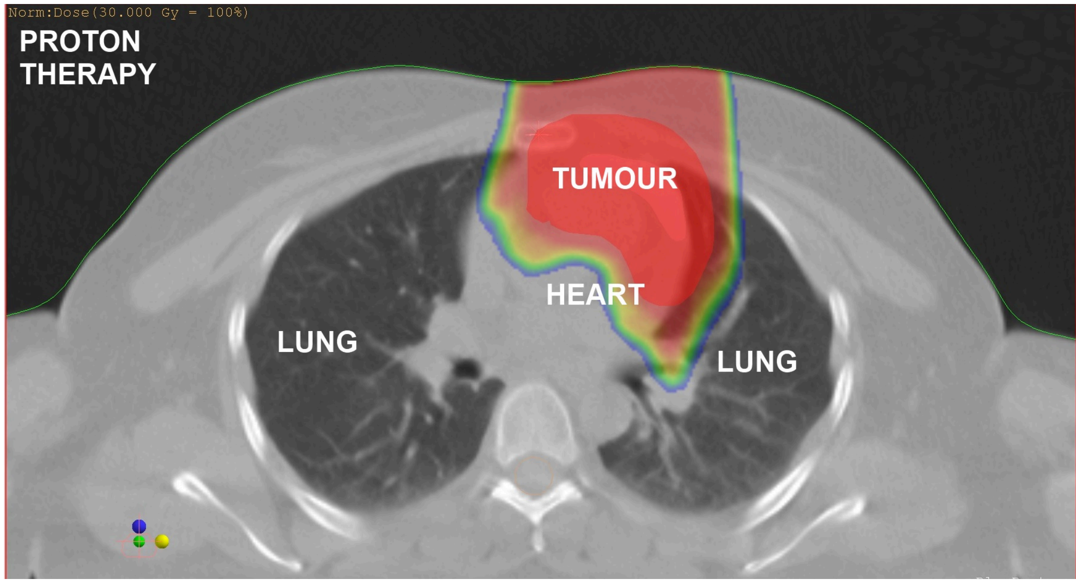






Advantage for tumor treatment





Higher irradiation of surrounding organs

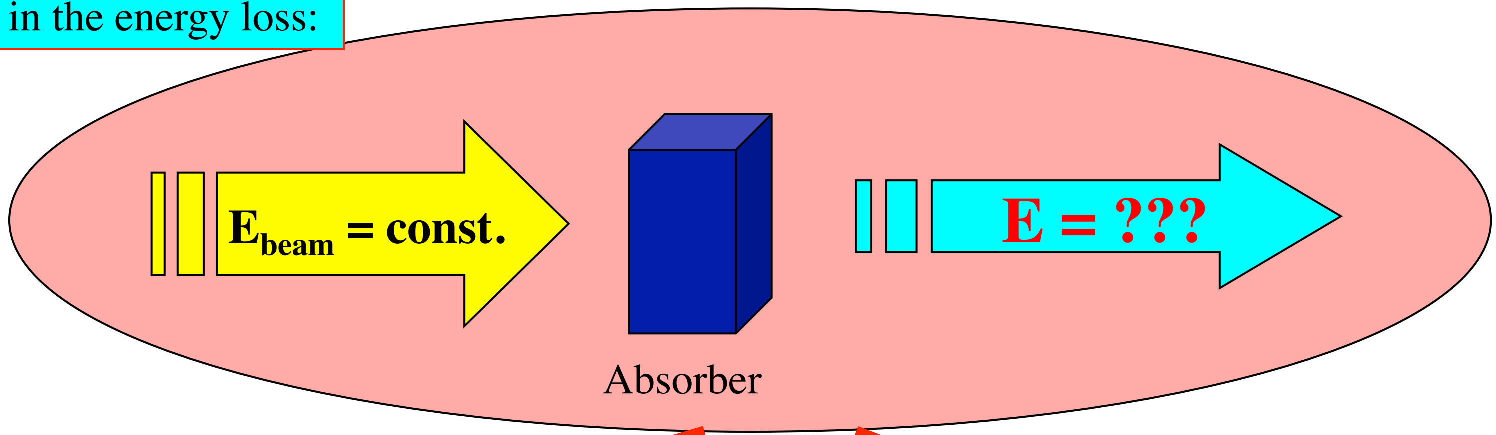
For Photons X et γ: $I = I_0 e^{-\mu x}$

With protons or heavy ions:

Heavy particles = less dispersion = better focused

dE/dx →Bragg peak → energy concentrated in tumor cells

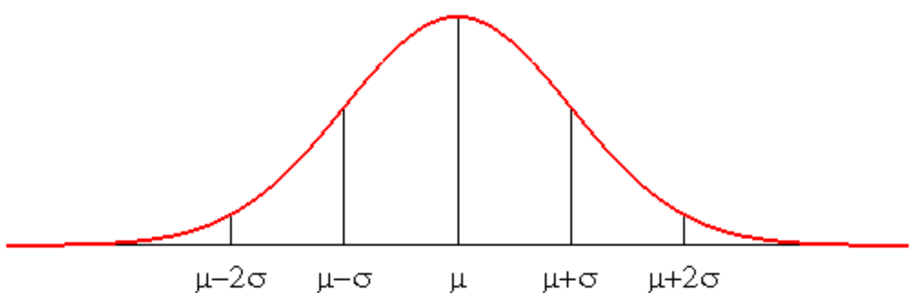
Fluctuations in the energy loss:



Thick Absorber:

Large number of collisions

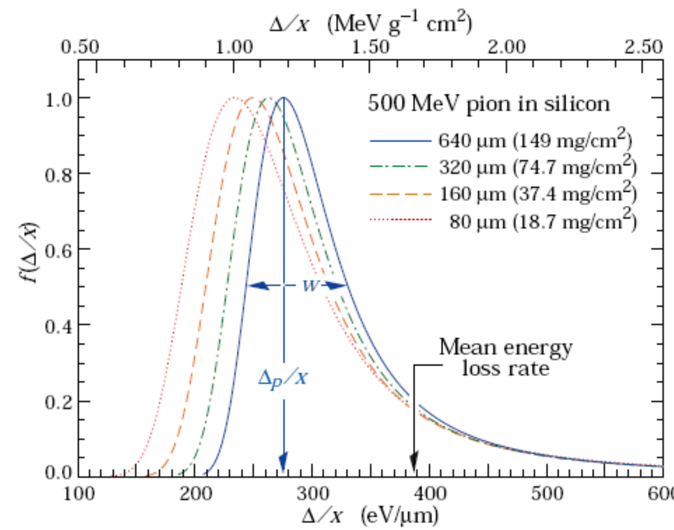
→ **Gauss**



Thin Absorber:

Small number of collisions

→ **Landau Distribution**



δ-Rays

- Energy loss distribution is not Gaussian around mean.
- In rare cases a lot of energy is transferred to a single electron

δ-Ray

- If one excludes δ-rays, the average energy loss changes
- Equivalent of changing E_{max}

Electrons

Energy loss of Electrons and Positrons

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{rad} + \left(\frac{dE}{dx}\right)_{coll}$$

1. Energy loss by ionization like heavy particles:

Dominant at energies < 20 MeV

Bethe-Bloch Equation for electrons:

$$\left(\frac{dE}{dx}\right) = 0{,}307\left(\frac{MeV}{g/cm^2}\right)\frac{Z}{A}\rho\frac{1}{\beta^2}\left(\ln\frac{2T(T+2m_e)}{I\times m_e} - \beta^2\right)$$

T = Kinetic energy of the electron
I = Ionization potential

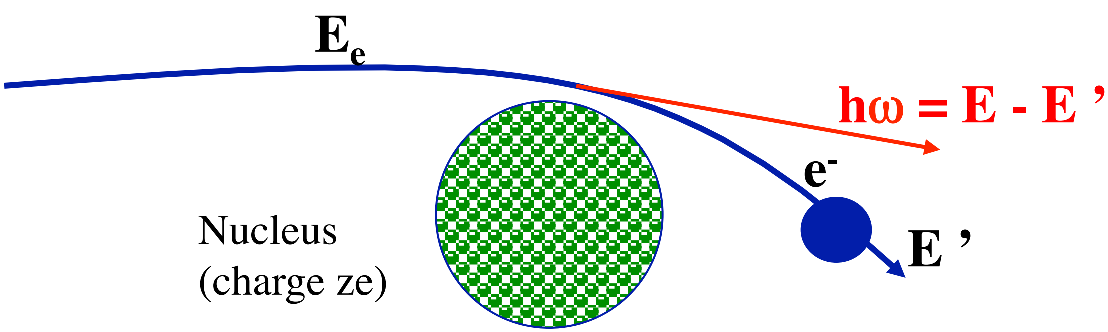
Two modifications needed in the equation

- Small mass —> larger deviation of the trajectory
- Diffusion of two identical particles (Pauli)

Energy loss of Electrons and Positrons

2. Energy loss by radiation (Bremsstrahlung): For E > 20 MeV

Classical interpretation::
Radiation from the acceleration of an electron or positron in the field of the nucleus.



$$\frac{d\sigma}{dk} \cong 5\alpha z^2 Z^2 \left(\frac{m_e c^2}{Mc^2\beta}\right)^2 \frac{r_e^2}{k} \ln\left(\frac{Mc^2\beta^2\gamma^2}{k}\right)$$

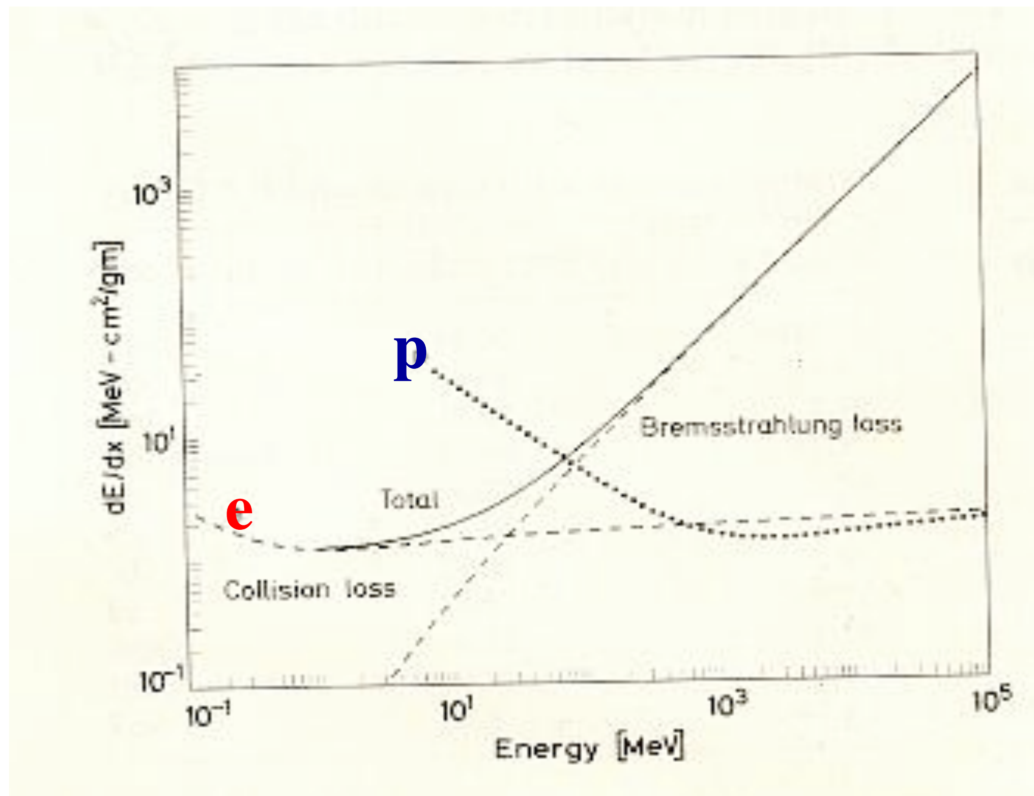
With k = Energy of the radiation (photons)

Energy loss of Electrons and Positrons

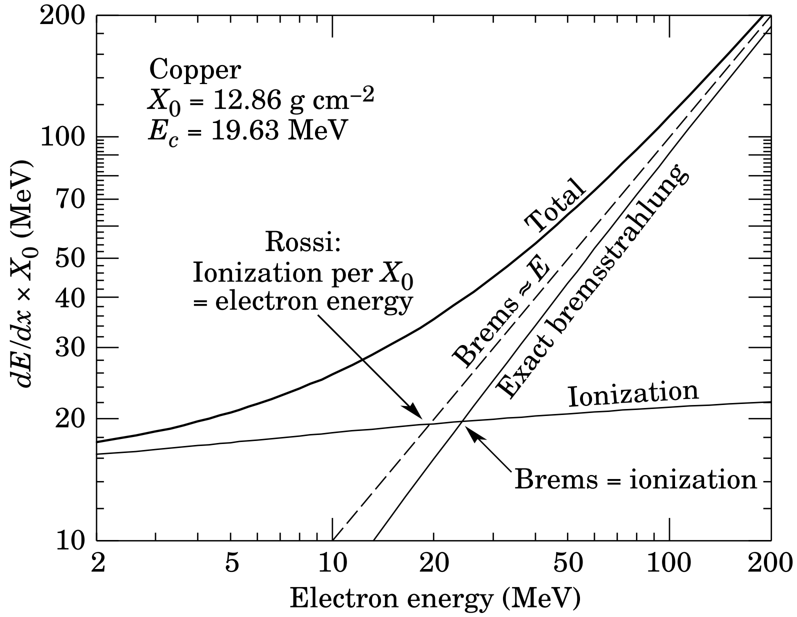
$$\frac{d\sigma}{dk} \propto \frac{1}{M_{\text{incomming particle}}}$$

- For a muon (M = 106 MeV) σ_{brems} is **40000** times smaller than for an electron!
- For a proton σ_{brems} is roughly **4 million** times smaller!!!

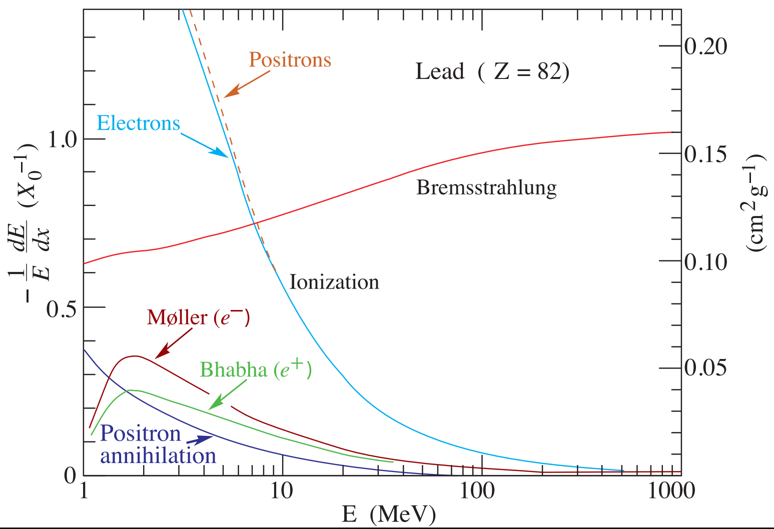
In first order, Energy loss by Bremsstrahlung is only relevant for electrons



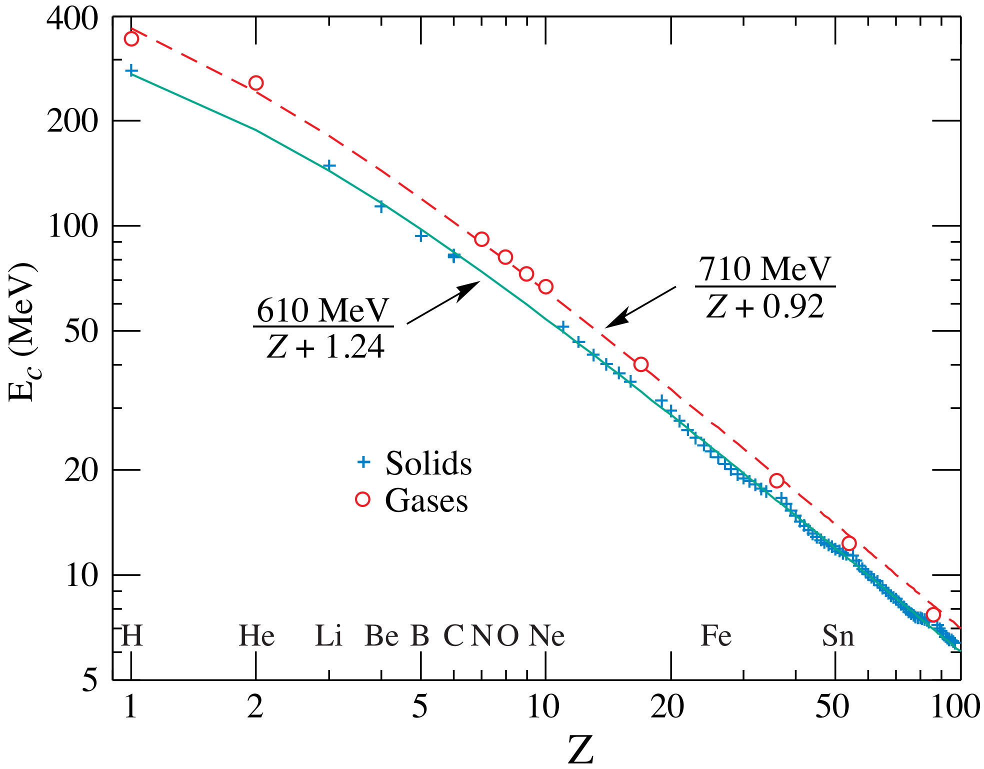
Definition of radiation length: X_0 = Average distance traveled by an electron before loosing 1/e of its energy by Bremsstrahlung.



Fractional energy loss per radiation length:



E_c [Critical Energy] Brems = Ionization



Range of Electrons

Multiple scattering in matter:

The range is very different from the dE/dx by Bethe-Bloch

Differences from 20% to 400%

More fluctuations in dE/dx than for heavy particles:

1. Energy transfer in each collision is bigger
2. Bremsstrahlung

Some empirical formulas to calculate the range of electrons::

Sternheimer relation:

$$\mathbf{R_e(T) = (0.486\ g\ cm^{-2})\ T^n}$$

with $n = 1.265 - 0.954 \ln(T)$

T en MeV

Example: Electron with T = 100 KeV in a TPC
With He at 77 K and 5 bars:
$R(T) = (0.486\ g\ cm^{-2} / 3{,}124 \times 10^{-3}\ g\ cm^{-3})\ T^{(1{,}265\ -\ 0{,}0954\ \ln(0{,}1))}$
R(0,1MeV) = 5 cm

Range of Electrons

$$R(T) = A \times E\left(1 - \frac{B}{1 + CT}\right)$$

(Valid for small and medium Z)

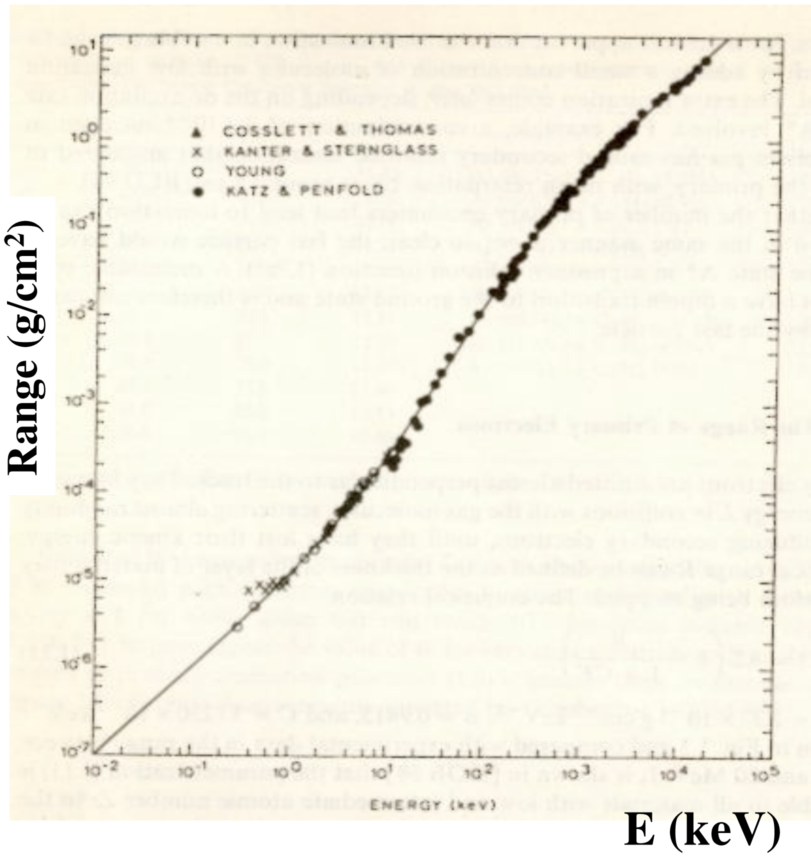
Avec: $A = 5.37 \times 10^{-4}$ g cm^{-2} KeV^{-1}

$B = 0.9815$

$C = 3.1230 \times 10^{-3}$ KeV^{-1}

300 eV < T < 20 MeV

Blum, Rolandi:Particle Detection with Drift Chambers
Springer Verlag, 1993

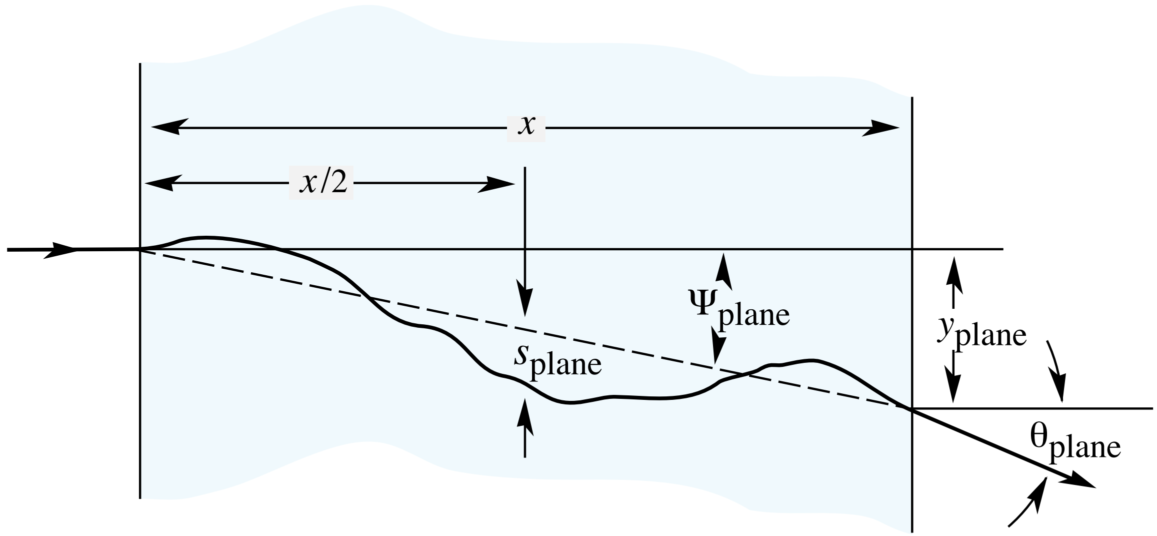


Multiple scattering through small angles

- Charged particles traversing a medium are deflected by many small angle scatters.
- Scattering is mostly due to Coulomb scattering from nuclei. (for hadrons strong interaction also contributes)
- Angular distribution described by Molière theory and is in first approximation Gaussian.
- For large angles = Rutherford scattering (larger tails than the Gaussian distribution).

Gaussian approximation:

$$\theta_0 = \theta_{plane}^{rms} = \frac{1}{\sqrt{2}}\theta_{space}^{rms}$$



$$\theta_0 = \frac{13.6\mathrm{MeV}}{\beta c \times p} z\sqrt{x/X_0}\left(1+0.038\ln\{x/X_0\}\right)$$

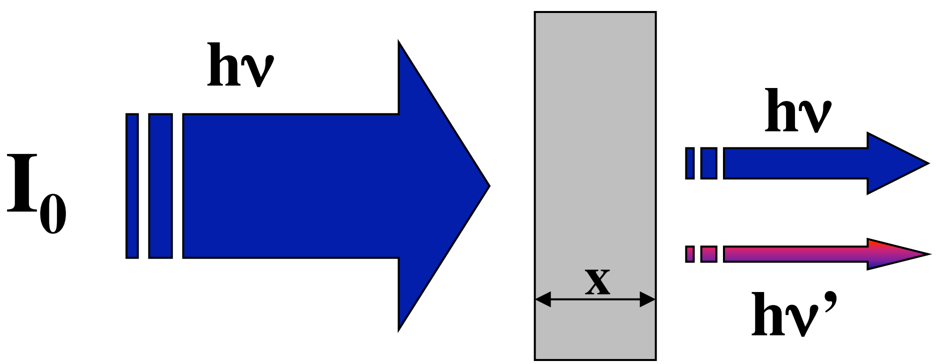
p, βc, z are momentum, velocity and charge of the incoming particle

Interactions of Photons

Interactions of Photons

Electric charge = 0 → **No Coulomb scattering with electrons of matter**

- Deeper penetration in matter (smaller cross section)
- A beam of photons traversing a slab of matter is attenuated in **intensity**, NOT in energy!
- Beam photons which passed through did NOT undergo an interaction.
- If they had an interaction, they change energy.



$$I(x) = I_0 \exp(-\mu x)$$

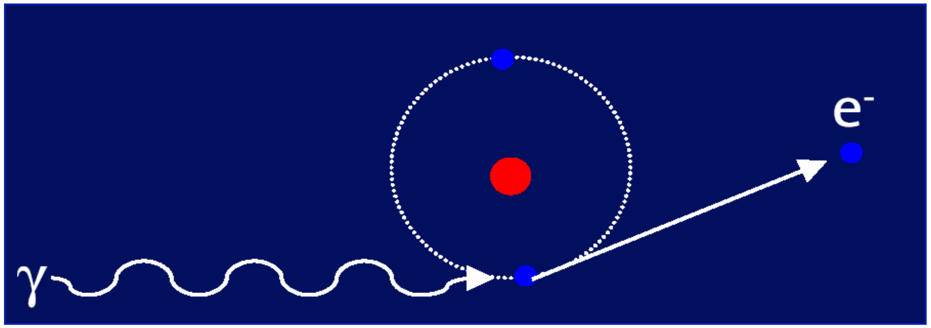
With: I_0 = Intensity of the beam
μ = photon absorption coefficient
x = path length

Interactions of Photons

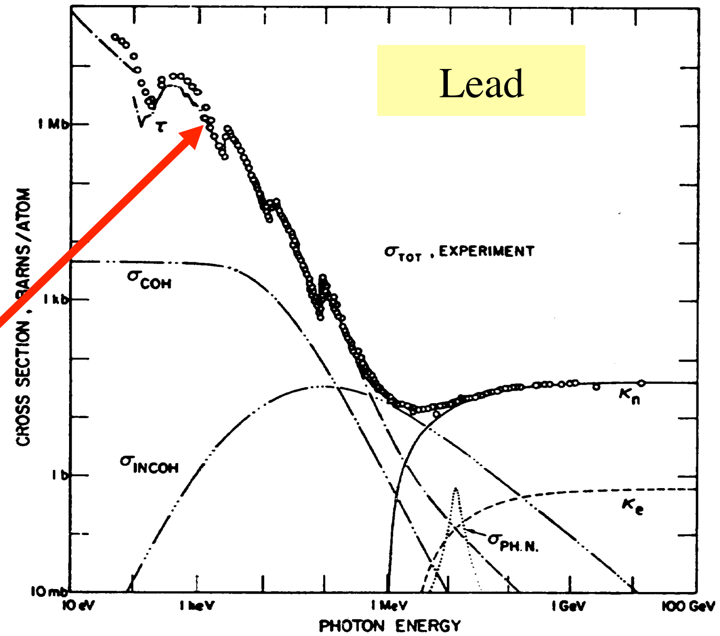
Three dominant Interactions:

1. Photoelectric effect: **absorption of the photon, ejection of the electron**

$$E_{(electron)} = h\nu - E_{binding}$$

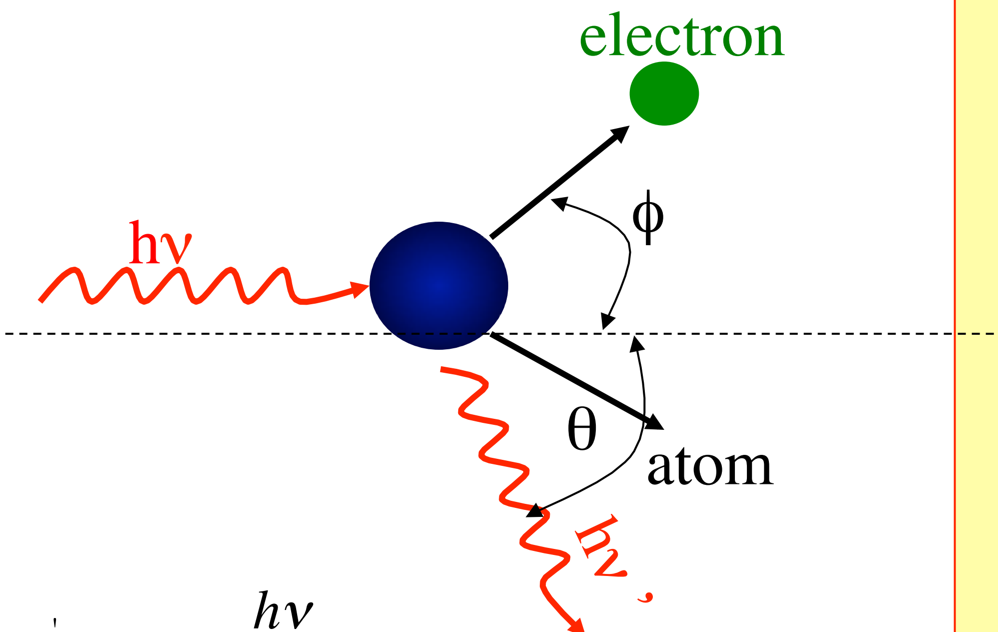


$$\sigma_{ph} = 4\sqrt{2}\alpha^4 Z^5 \left(\frac{8\pi r_e^2}{3}\right)\left(\frac{m_e c^2}{h\nu}\right)^{\frac{7}{2}}$$



Einstein: Prix Nobel 1921 pour l'explication de l'effet photoélectrique

2. Compton Scattering: **elastic scattering on a free electron**

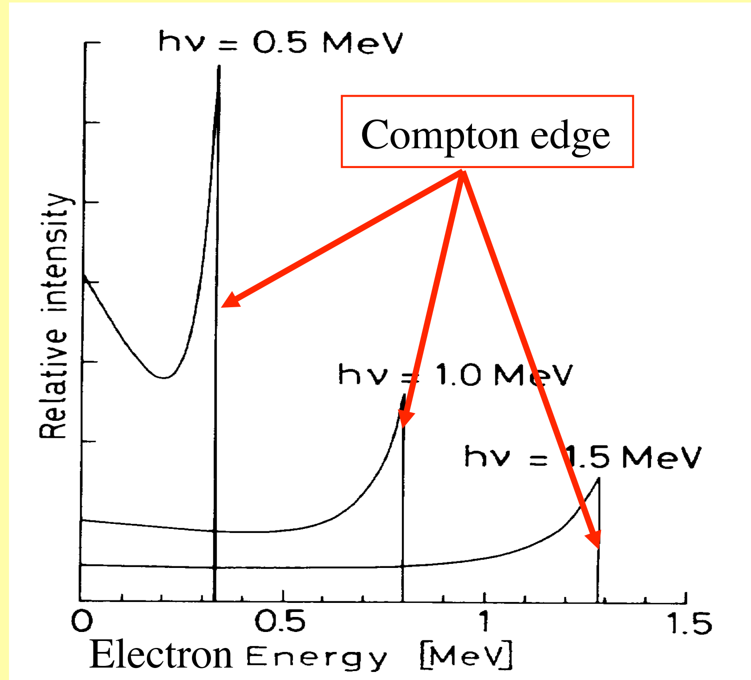


$$h\nu' = \frac{h\nu}{1+\gamma(1-\cos\theta)}$$

$$T = h\nu - h\nu' = h\nu \frac{\gamma(1-\cos\theta)}{1+\gamma(1-\cos\theta)}$$

$$\cot\varphi = (1+\gamma)\tan\frac{\theta}{2}, \gamma = h\nu / m_e c^2$$

Energy distribution of Compton recoil electrons:



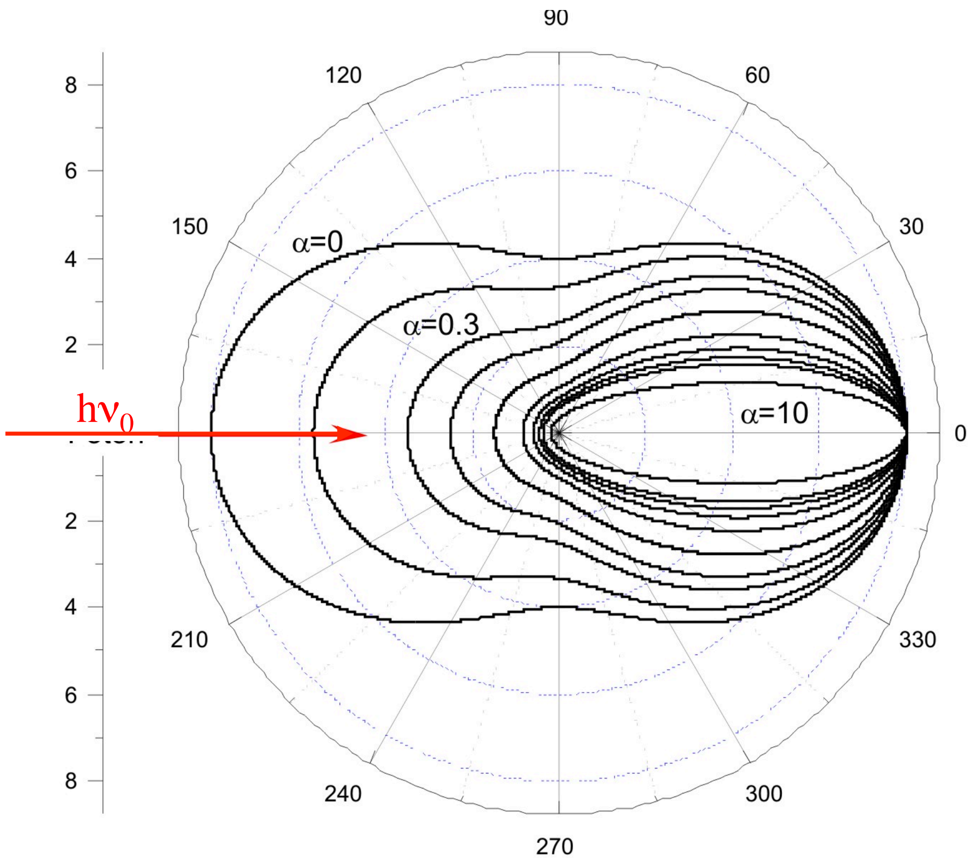
$$T_{\max} = E_{\gamma,in} \frac{2\gamma}{1+2\gamma}$$

Compton scattering:

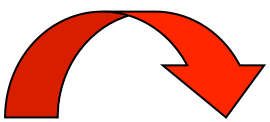
Angular distribution of the scattered photon

$\alpha = h\nu_0 / m_0c^2$

$m_0c^2 = 0.511$ MeV



α large

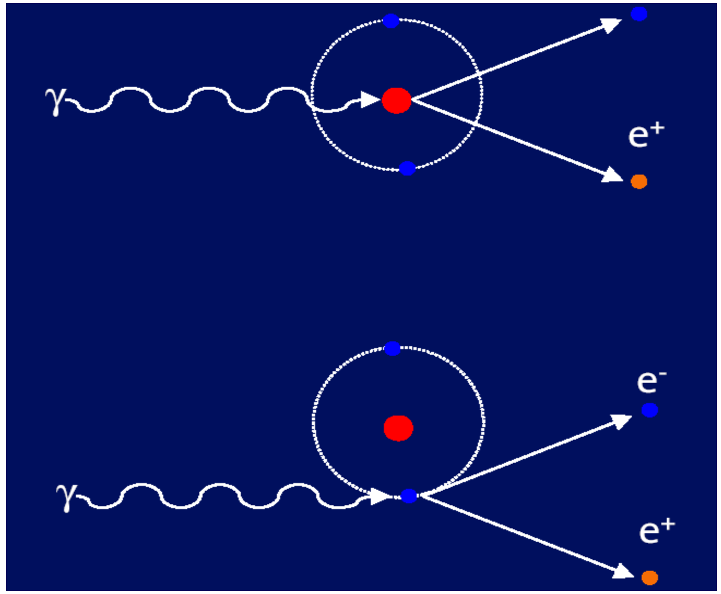


Photons scattered in forward direction

Interactions of Photons

3. Pair Production: absorption of the photon and creation of a pair electron - positron

Creation in the field of the nucleus



Creation in the field of the electron

$$E_{threshold} = 2m_ec^2 = 1{,}022 \text{ MeV}$$

At high energies ($E_\gamma >> 137 m_e c^2 Z^{-1/3}$) the pair production cross section is almost constant

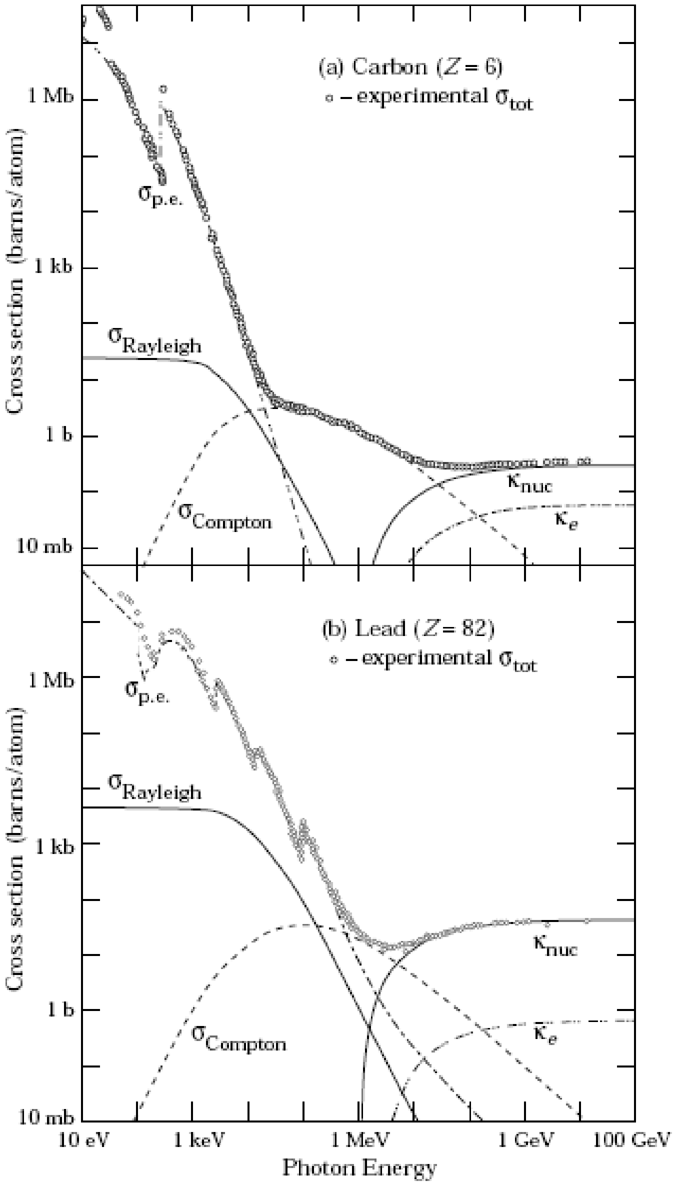
$$\sigma_{paire} = 4Z^2 \alpha r_e^2 [7/9\{\ln(183 Z^{-1/3}) - f(Z)\} - 1/54]$$

f(z) = correction à l 'approximation de Born pour
L 'interaction coulombienne d 'électron dans
le champ électrique du noyau

$$\sigma = \frac{7}{9}\left(\frac{A}{X_0 N_A}\right)$$

For E > 1 GeV and high Z

Cross Sections for Photon Interactions:



$\sigma_{p.e.}$ = effet photo-électrique atomique
(absorption du photon, émission d 'un électron)

$$\sigma_{pe} \approx Z^5$$

$\sigma_{coherent}$ = diffusion cohérente
(diffusion Rayleigh - ni ionisation, ni excitation d 'atome tout les électrons d 'atome en contribution les photons ne perdent pas d 'énergie)

$\sigma_{incoherent}$ = diffusion incohérente
(diffusion Compton sur un électron)

$$\sigma_{compton} \approx Z$$

σ_{nuc} = absorption nucléaire

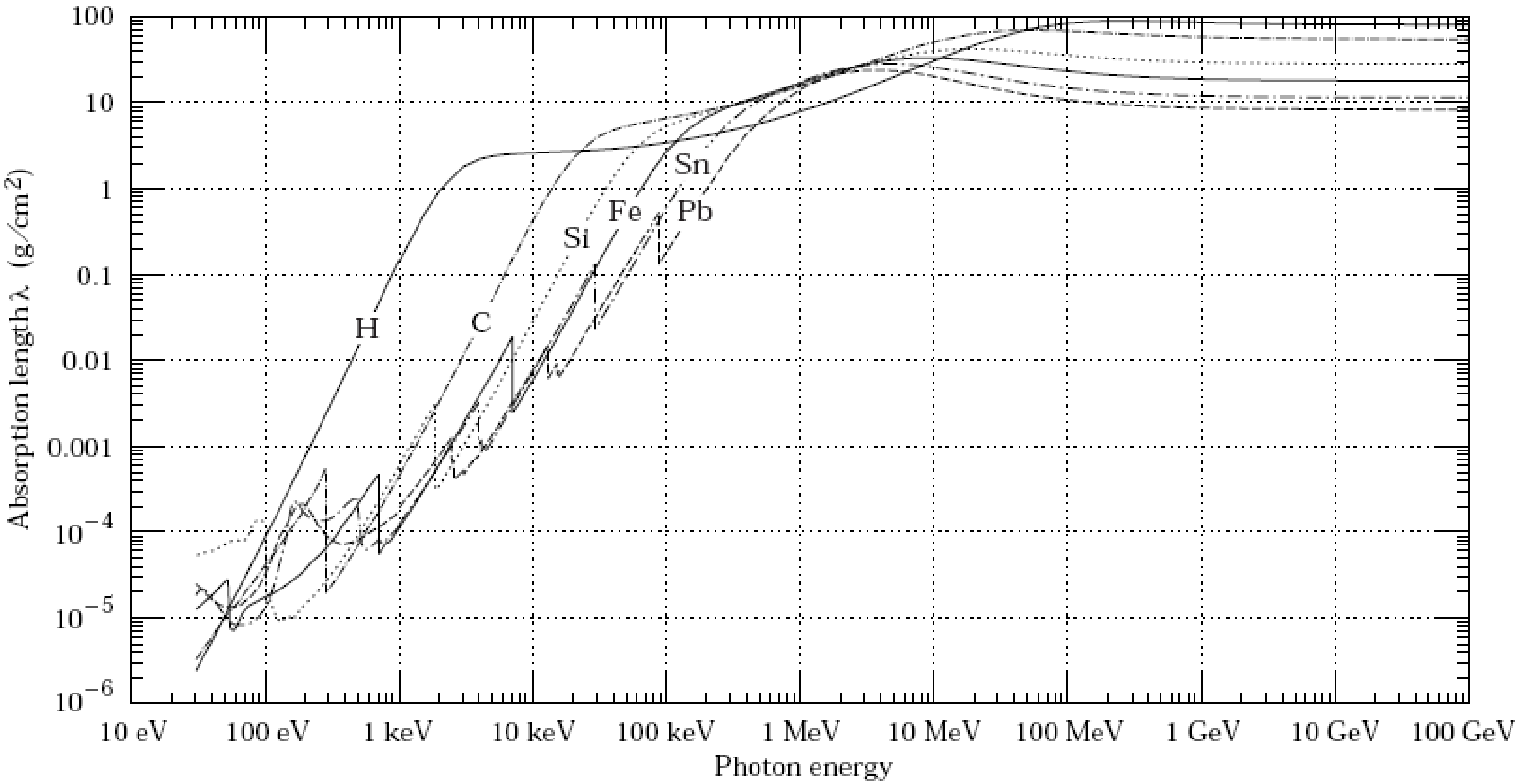
κ_n = production paire dans champ nucléaire

κ_e = production paire dans champ électronique

$$\sigma_{pair} \approx Z^2$$

Interactions of Photons

Photon mass attenuation length (mean free flight path)



$$\lambda = \frac{1}{\mu/\rho}$$ where μ/ρ is the mass attenuation coefficient, ρ = density

Electromagnetic Cascades:

High Energy Photon or Electron

↓

Pair production and Bremsstrahlung

↓

Generation of many electrons and photons of lower energy

↓

Pair production and Bremsstrahlung

↓

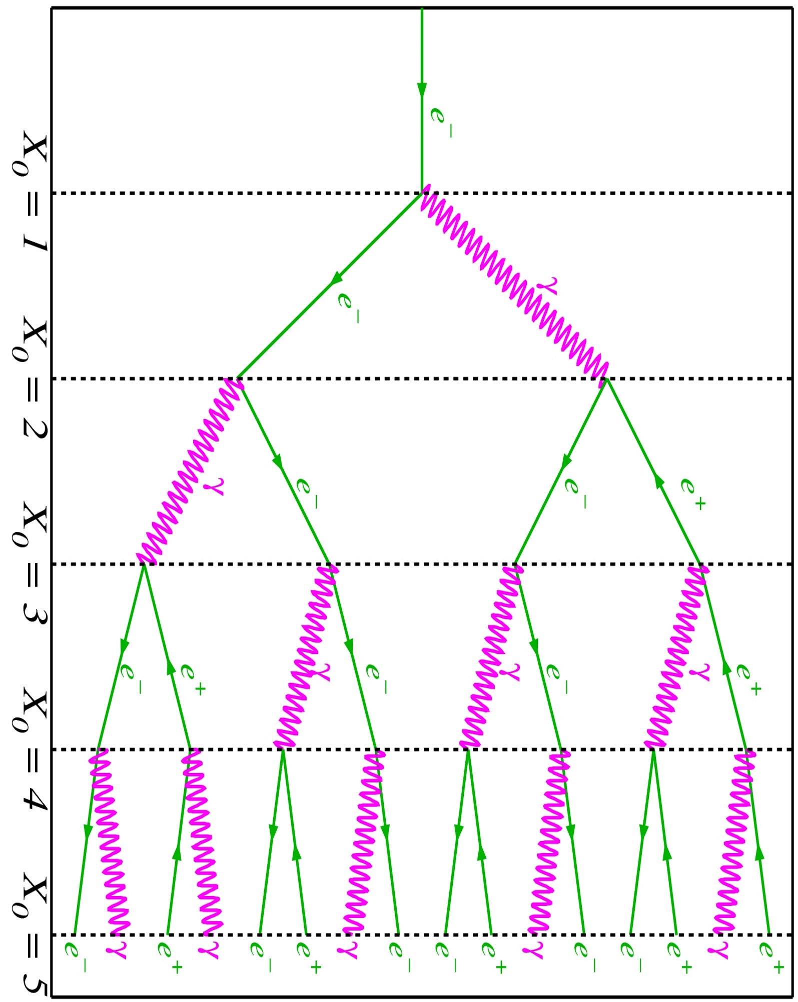
Generation of many electrons and photons of lower energy

↓

$E = E_{critique}$

↓

Cascade stops → dE/dx by ionization



Electromagnetic Cascades:

Some simple approximation:

1/ Longitudinal developement:

An interaction occurs after each radiation length, after t radiation lengths we have a total of
$N = 2^t$ particles
Each particle has an average energy of $E(t) = E_0 / 2$
Maximum penetration length of the cascade:
$E(t_{max}) = E_0 / 2^{tmax} = E_c$

$$t_{max} = \frac{\ln\left(\frac{E_0}{E_c}\right)}{\ln 2} \text{ and the maximum number of particles produced is } N_{max} \cong \frac{E_0}{E_c}$$

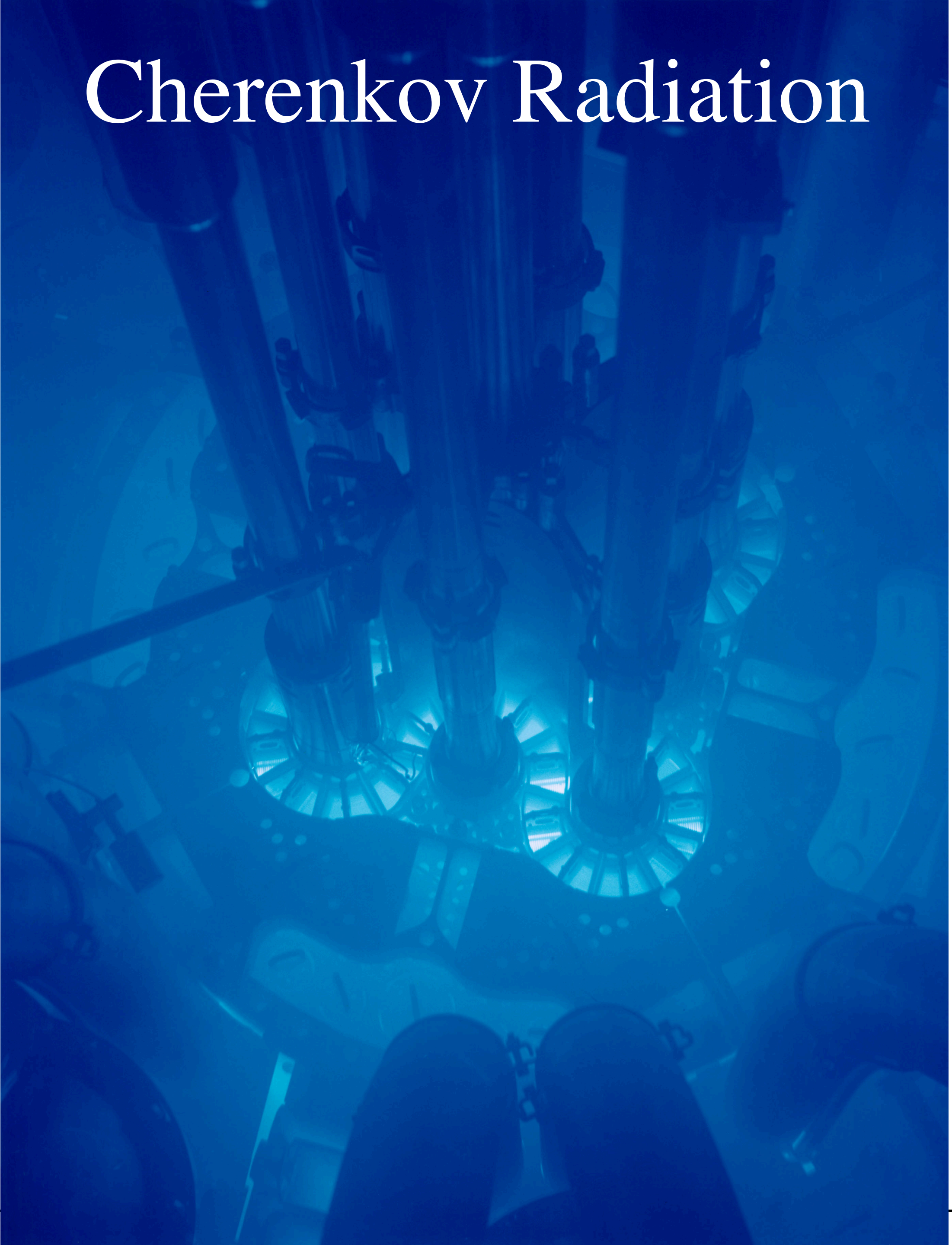
2/ Transversal dimensions:

$$\text{Molière radius}: R_M = X_0 \frac{E_s}{E_c} \quad \text{with } E_s = \sqrt{4\pi/\alpha} \times m_e c^2 = 21\,\text{MeV (scale energy)}$$

90% of the particles stay inside a cylinder with R_M around the shower axis.

Cherenkov Radiation

Cherenkov Radiation



Pavel Alekseyevich Cherenkov

1904-1990

Physics Institute of USSR Academy of Sciences, Moscow

Nobel prize in 1958

Cherenkov Radiation

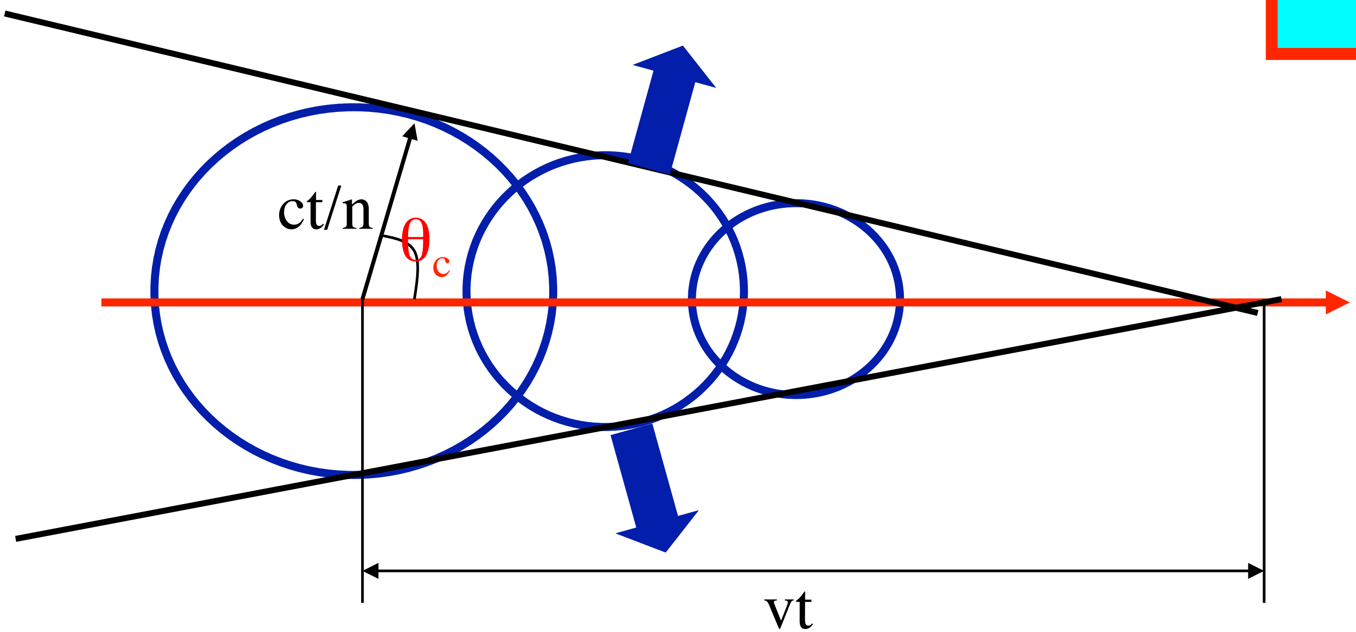
A particle goes faster than the speed of light in the material

Emission of Cherenkov radiation

ßc = v = c/n, n = index of refraction of the medium

Condition: $v_{part} > c/n$

$$\cos\theta_c = \frac{ct/n}{\beta ct} = \frac{1}{\beta n}$$



Cherenkov Radiation

θ_c = Cherenkov angle
Radiation of « Cherenkov » photons with a continues spectrum
The photons are polarized

Firs theory by Tamm et Frank (Prix Nobel with Cherenkov)

$$\left(-\frac{dE}{dx}\right)_{\text{Cherenkov}} = \frac{4\pi e^2}{c^2}\int \omega d\omega \left(1 - \frac{1}{\beta^2 n^2}\right)$$

This is already included in the dE/dx by Bethe & Bloch (relativistic rise)

Energy loss by Cherenkov radiation: $-\left(\frac{dE}{dx}\right)_{\text{Cherencov}} \cong 10^{-3}\ \text{MeVcm}^2\text{g}^{-1}$

Energy loss by collision in H_2: $-\left(\frac{dE}{dx}\right)_{Coll} \cong 0{,}1\ \text{MeVcm}^2\text{g}^{-1}$

Energy loss by collision in a gas with large Z: $-\left(\frac{dE}{dx}\right)_{Coll} \cong 0{,}01\ \text{MeVcm}^2\text{g}^{-1}$

Cherenkov Radiation

Number of Cherenkov photons per path length of a particle of charge ze and per unit of photon energy:

$$\frac{\mathrm{d}^2 N}{dE dx} = \frac{\alpha^2 z^2}{r_e m_e c^2}\left(1 - \frac{1}{\beta^2 n^2(E)}\right)$$

$\approx 370 \sin^2\theta_c(E)\ \mathrm{eV}^{-1}\ \mathrm{cm}^{-1}$ (with $z = 1$)

For photons of $400\ \mathrm{nm} < \lambda < 700\ \mathrm{nm}$ ⇒ **$N/L \approx 490 \sin^2\theta_c$**

Example: How to build a huge Water Cherenkov detector?

Question: Should one use a normal window or Silica for the PM?

$$\frac{dN}{dx} = 2\pi z^2 \alpha \sin^2\theta \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2} \qquad \text{avec} \qquad 2\pi z^2 \alpha = 4{,}584\times 10^{-2}$$

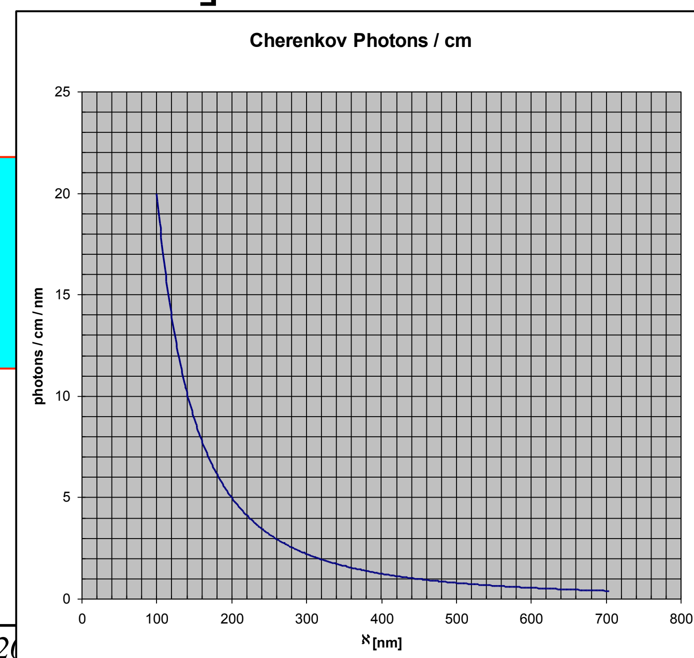
Pour H_2O: n = 1.33

$\cos\theta = 1/\beta n$, avec $\beta = 1$: $\theta = 41.25°$

$\sin^2\theta = 0.437$

$$\frac{dN}{dx} = 2\pi z^2 \alpha \sin^2\theta \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2} = 2\times 10^{-2}\left[\frac{1}{\lambda_{min}} - \frac{1}{\lambda_{max}}\right] [\text{photons/nm}]$$

$$\frac{dN}{dx} = 2\times 10^{5}\left[\frac{1}{\lambda_{min}} - \frac{1}{\lambda_{max}}\right] [\text{photons/cm}] \quad (\lambda \text{ en nm})$$



For 180 nm – 550 nm: dN / dx = 747 photons / cm

For 380 nm – 550 nm: dN / dx = 303 photons / cm

Cherenkov Detectors

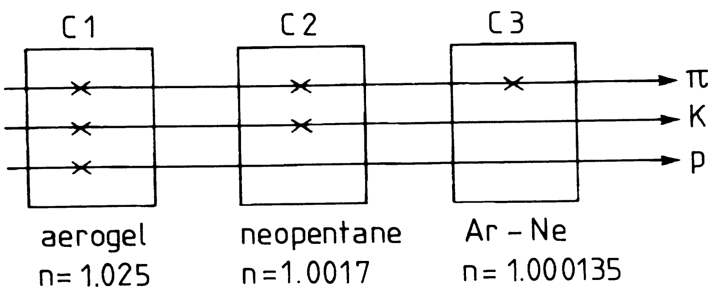
- **Threshold counter** (Yes / No)
- **Differential counter** (uses the Cherenkov angle)
- **Ring imaging counter** (uses the image of the Cherenkov ring)

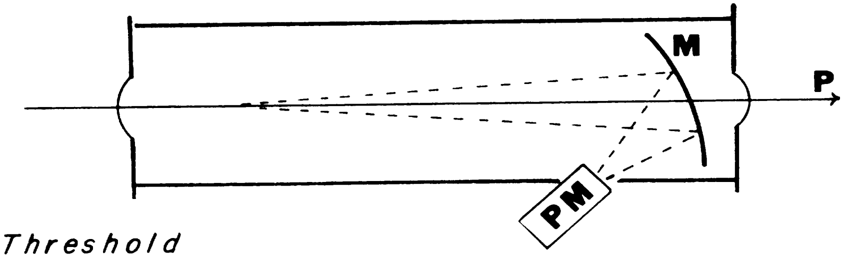
1. Threshold counter: Particle ID over threshold:

$$\beta_t = \frac{1}{n}$$

Example for He:

electrons	63 MeV/c
kaons	61 GeV/c
pions	17 GeV/c
protons	115GeV/c





Threshold

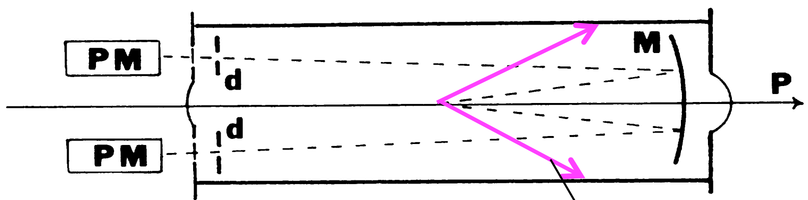
Cherenkov Detectors

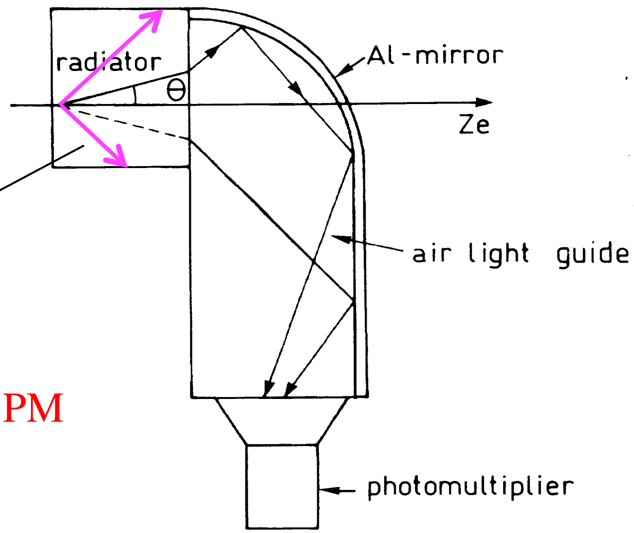
2. Différentiel counters: Emission of Cherenkov light at a defined angle:

For a given momentum, cosθ is fonction of the mass

$$\cos\theta = \frac{1}{n\beta} = \frac{1}{n(p/E)} = \frac{\sqrt{m^2 + p^2}}{np}$$

Used as beam monitor: e.g. contamination of π and k.

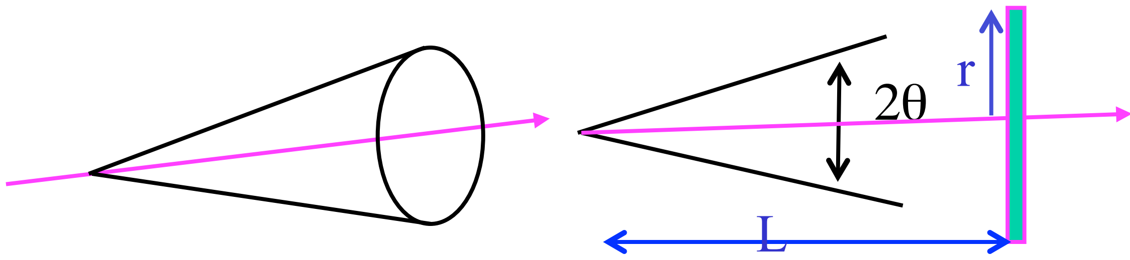


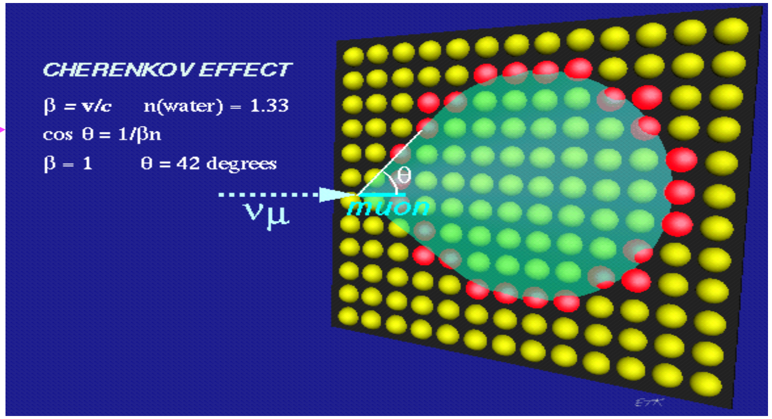


Only the light of a certain angle goes to the PM

Cherenkov Detectors

3. Ring imaging counter (RICH):





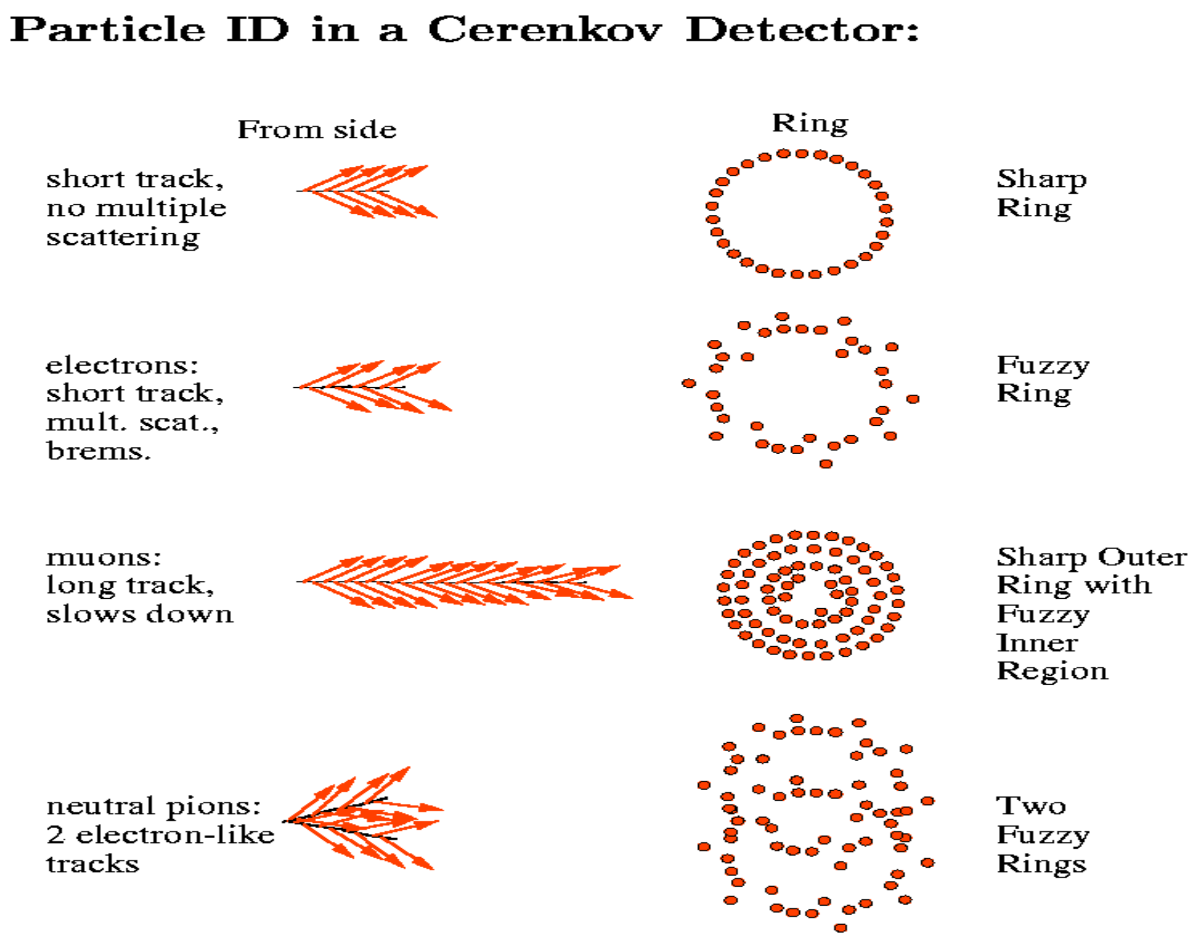
$r = L \times \tan\theta$

Incoming particle with p = 1GeV/c, L = 1 m, in LiF (n = 1.392):

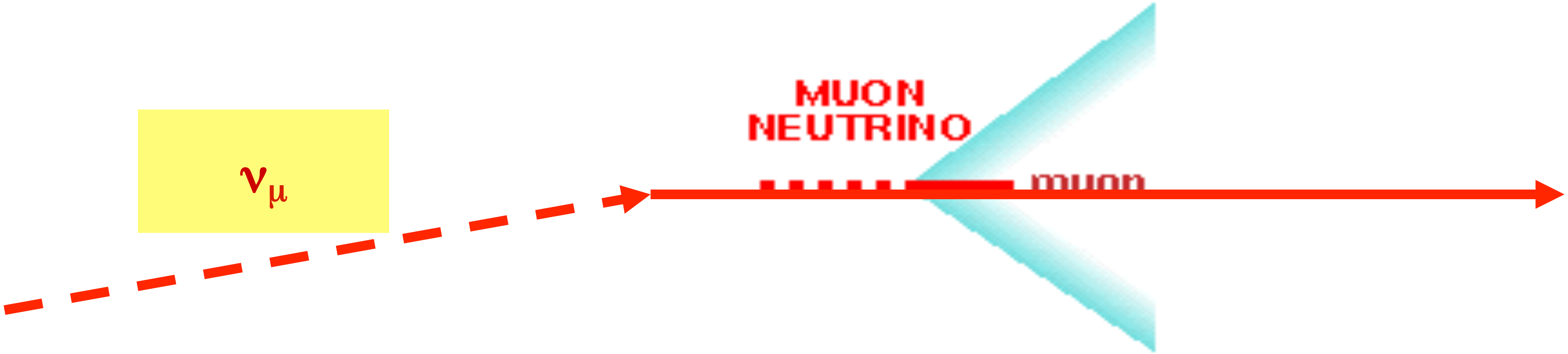
	θ(deg)	r(m)
π	43.5	0.95
K	36.7	0.75
P	9.95	0.18

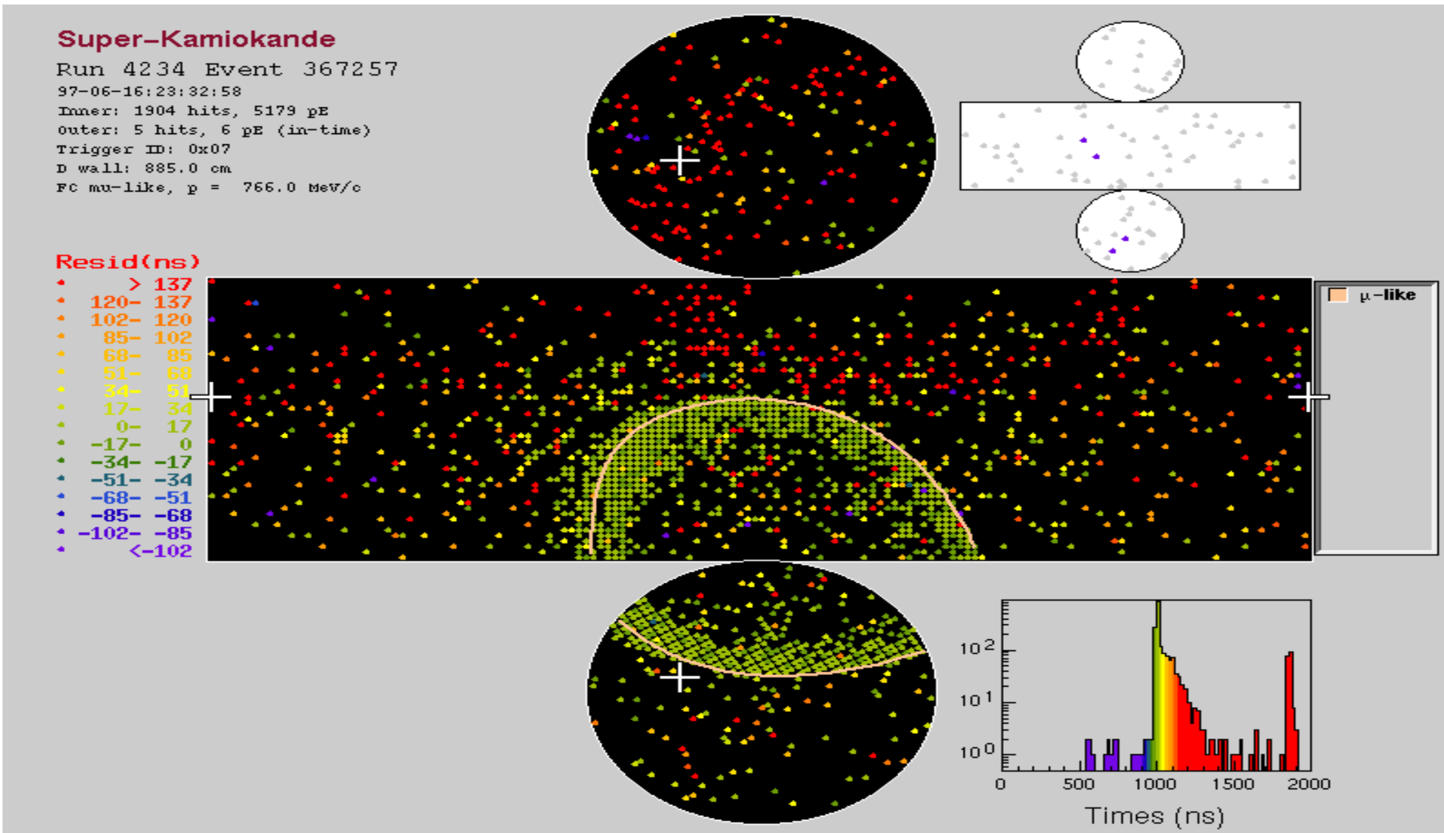
Very good π/K/p seperation

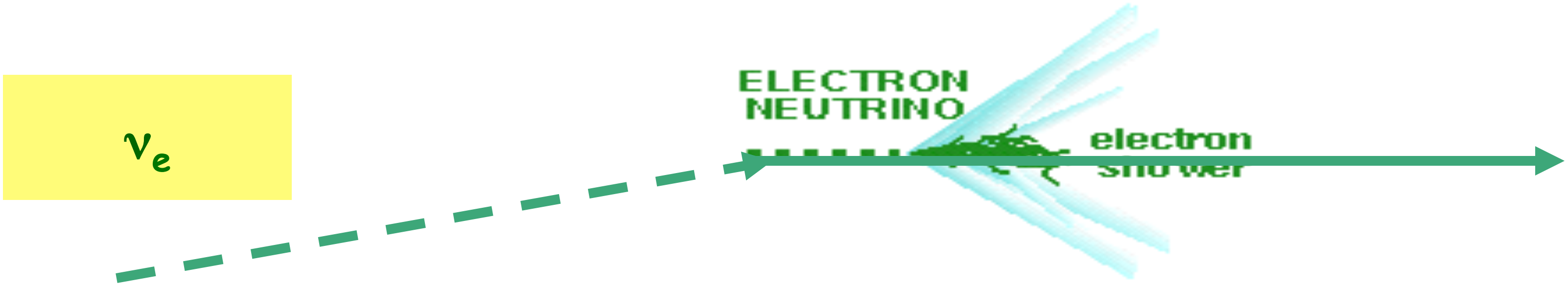
→ **Particle ID**

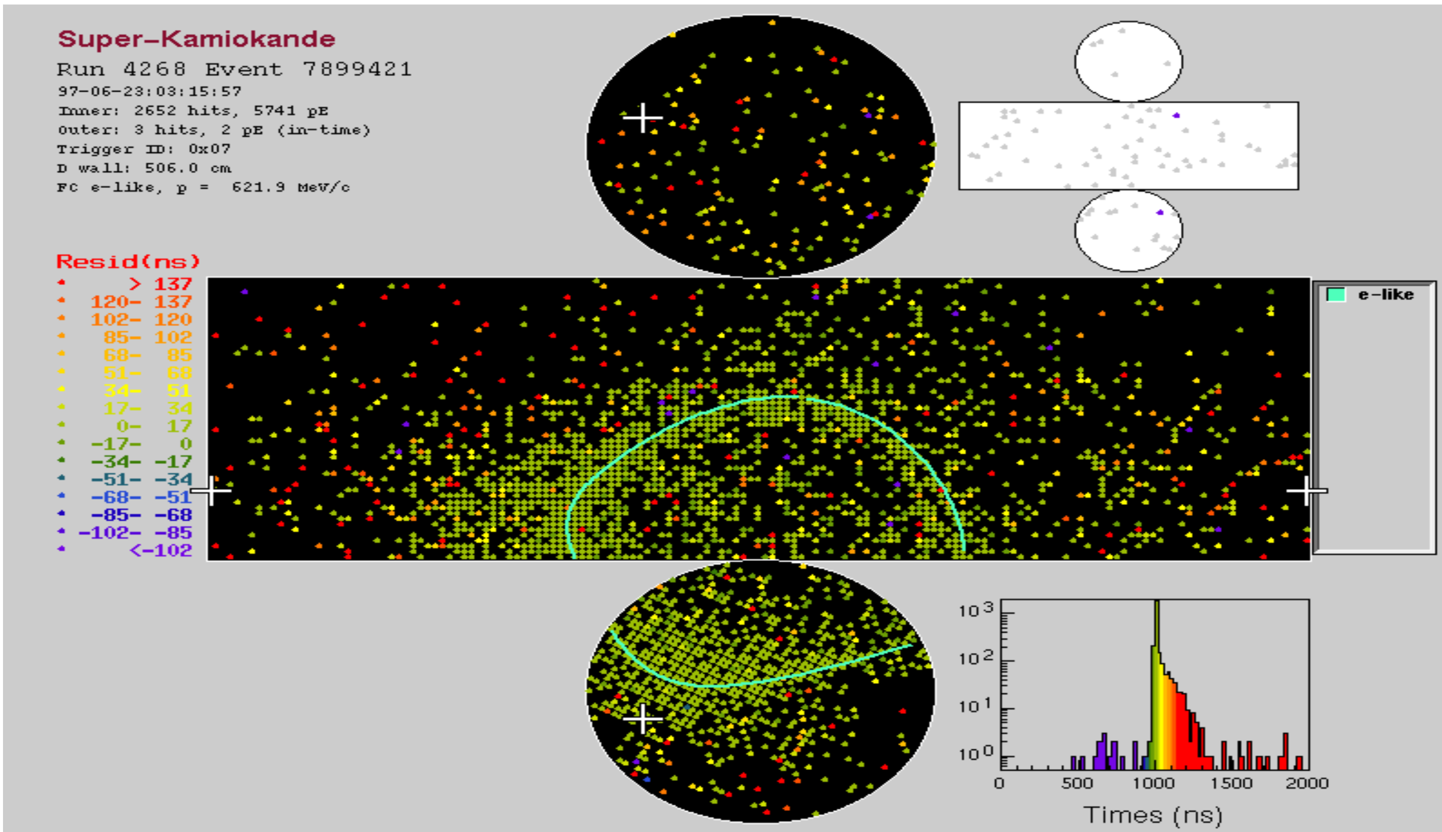


From SK and Miniboone)


ν_μ
MUON
NEUTRINO
muon

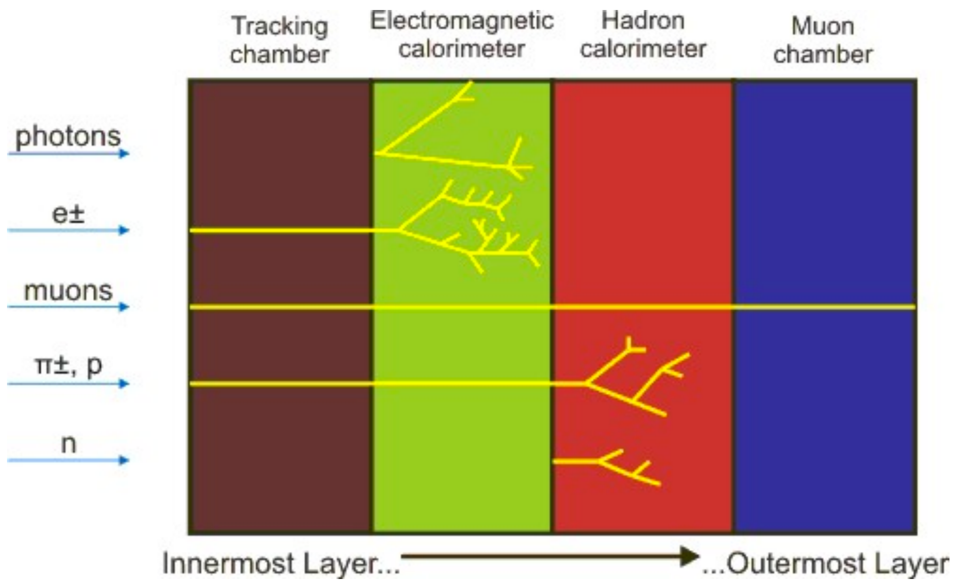

Super-Kamiokande
Run 4234 Event 367257
97-06-16:23:32:58
Inner: 1904 hits, 5179 pE
Outer: 5 hits, 6 pE (in-time)
Trigger ID: 0x07
D wall: 885.0 cm
FC mu-like, p = 766.0 MeV/c
Resid(ns)
> 137
120- 137
102- 120
85- 102
68- 85
51- 68
34- 51
17- 34
0- 17
-17- 0
-34- -17
-51- -34
-68- -51
-85- -68
-102- -85
<-102
μ-like
10^2
10^1
10^0
0
500
1000
1500
2000
Times (ns)


ν_e
ELECTRON
NEUTRINO
electron

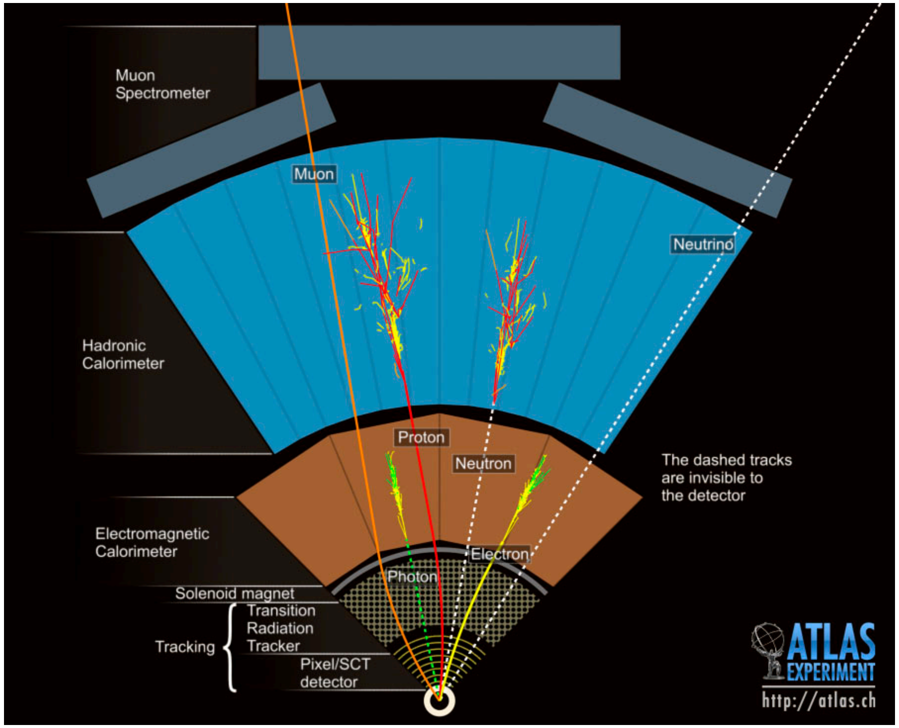

Super-Kamiokande
Run 4268 Event 7899421
97-06-23:03:15:57
Inner: 2652 hits, 5741 pE
Outer: 3 hits, 2 pE (in-time)
Trigger ID: 0x07
D wall: 506.0 cm
FC e-like, p = 621.9 MeV/c
Resid(ns)
> 137
120- 137
102- 120
85- 102
68- 85
51- 68
34- 51
17- 34
0- 17
-17- 0
-34- -17
-51- -34
-68- -51
-85- -68
-102- -85
<-102
e-like
10^3
10^2
10^1
10^0
0
500
1000
1500
2000
Times (ns)

Different interactions = different interaction length = typical HEP detector:

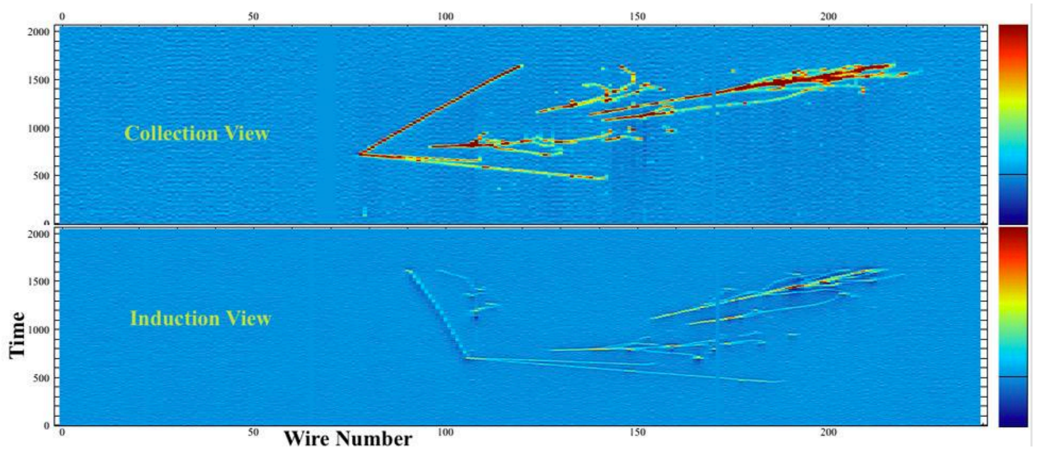
Generic detector:



Atlas:

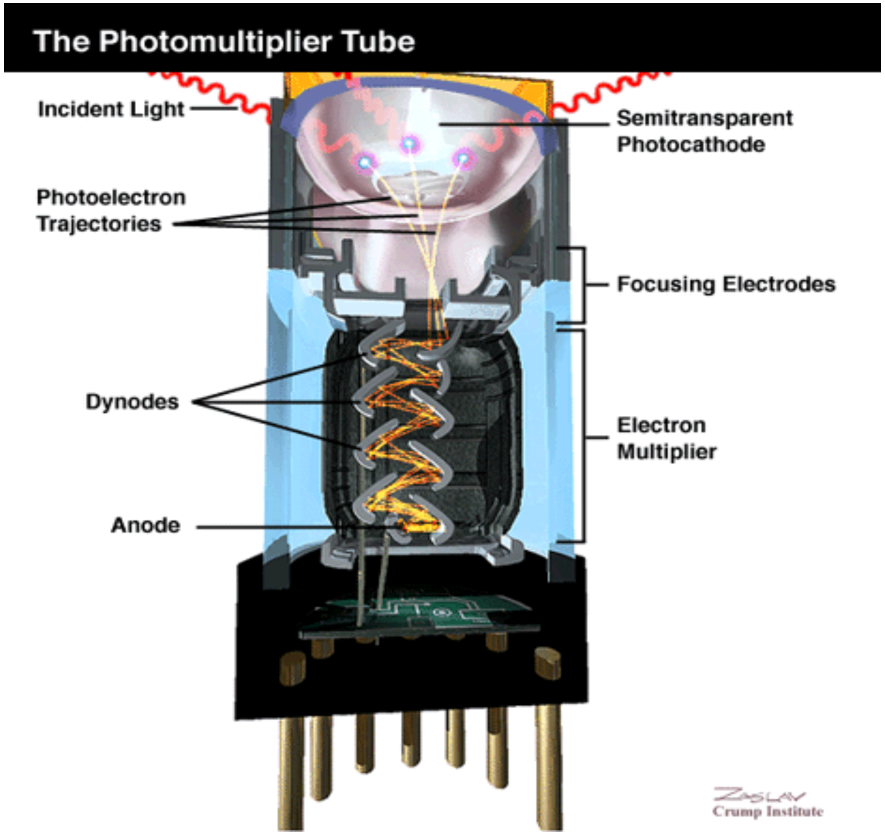


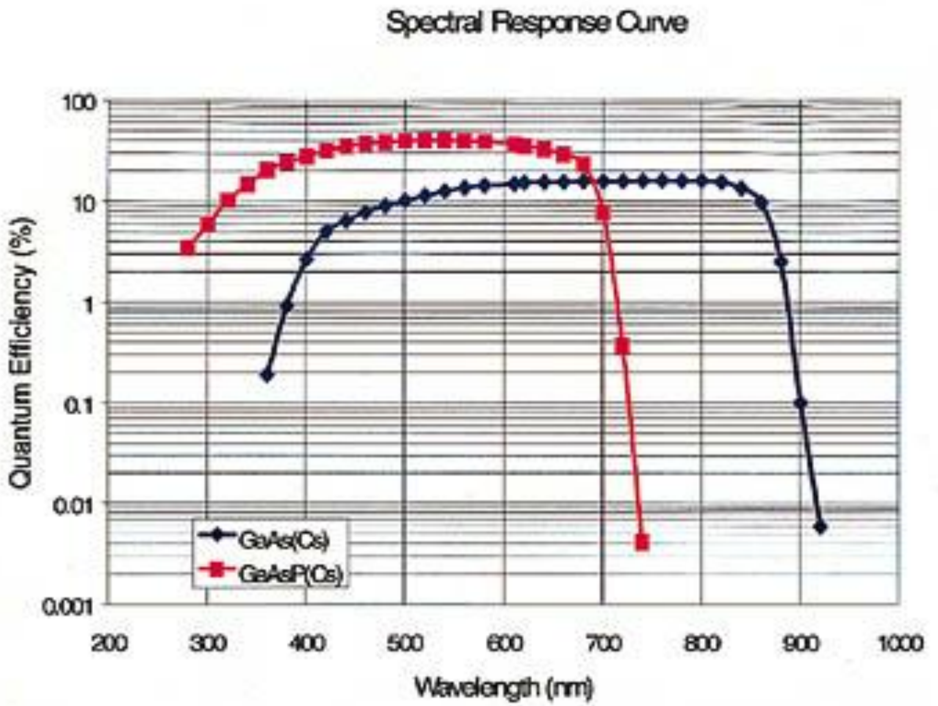
Neutrino event in a LAr detector:



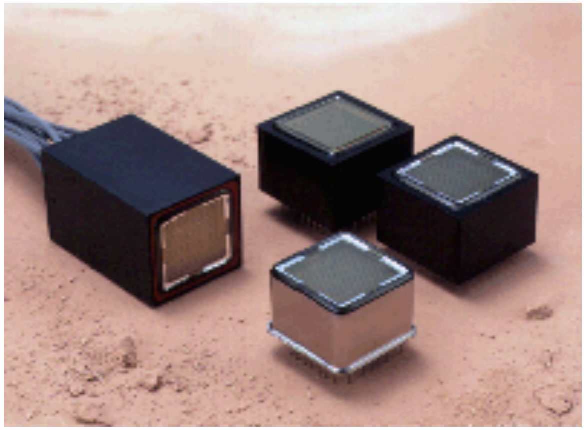
Some examples for light detection

Basic device: The Photomultiplier



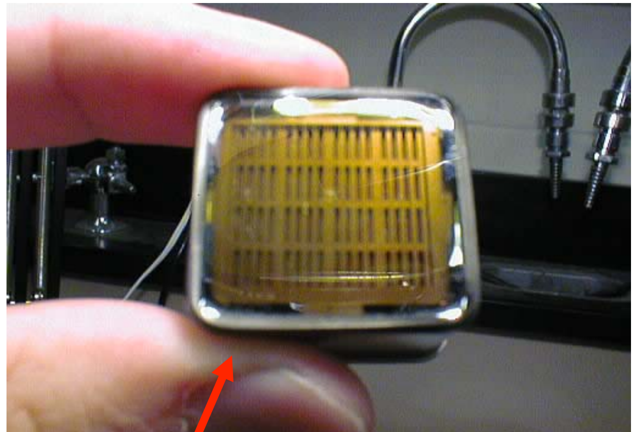


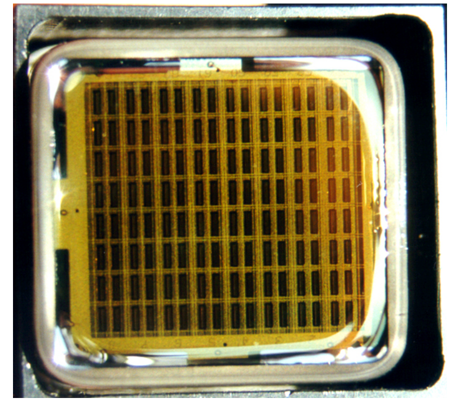
Multi-Anode Photomultiplier Tubes (MAPMT)



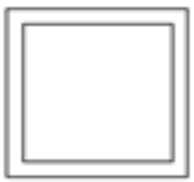
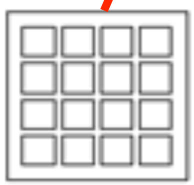
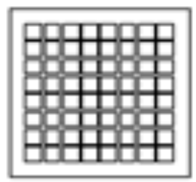
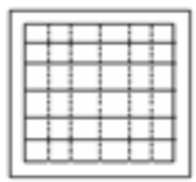
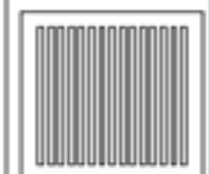
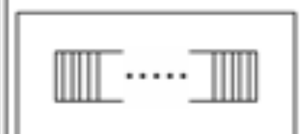
HV ≈ 900 V

M16





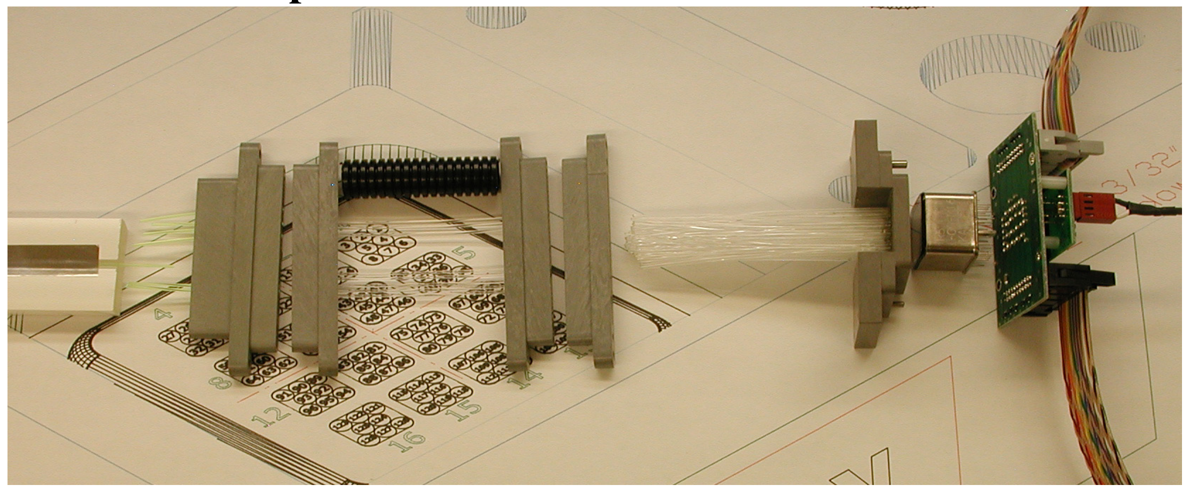
M64

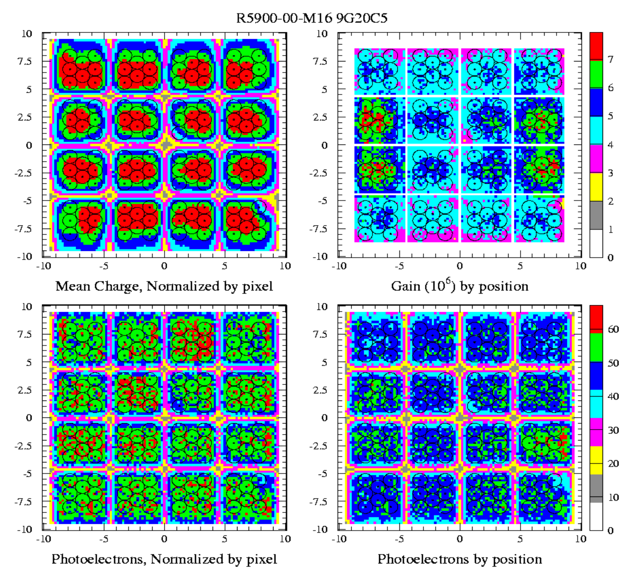
Type No.	R5900U	R5900U-00-M4	H6568 (R5900-00-M16)	H7546 (R5900-00-M64)	R8520-C12	R5900U-00-L16	H7260 (R7259)
Anode format							
Number of anodes	1	4	16	64	6(X)+6(Y)	16	32
Number of dynode stages	10	10	12	12	11	10	10

Multi-anode photomultiplicateurs (MAPMT)

Example: Test Hamamatsu M16 for the MINOS experiment:

1,2 mm WLS fibres and LED blue



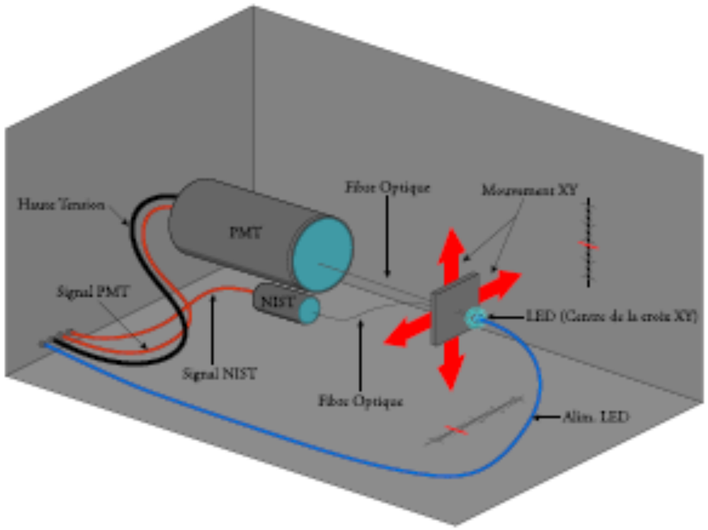


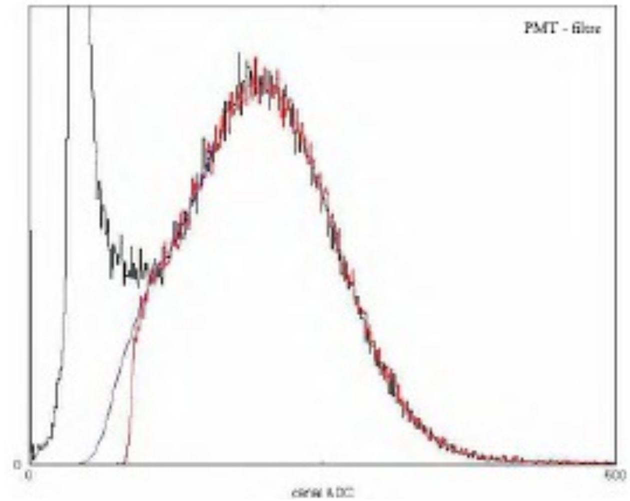
Attention to variation pixel to pixel but also inside one pixel

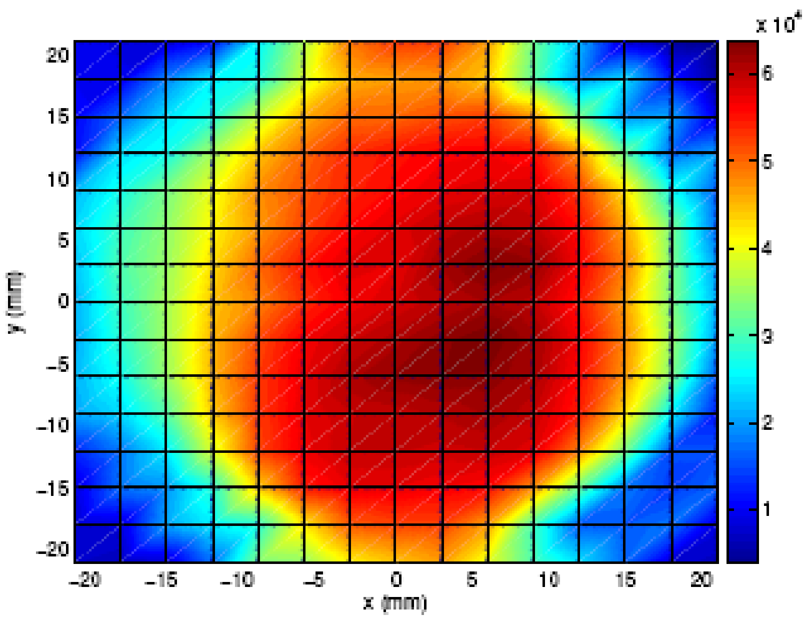
Variations up to 20%

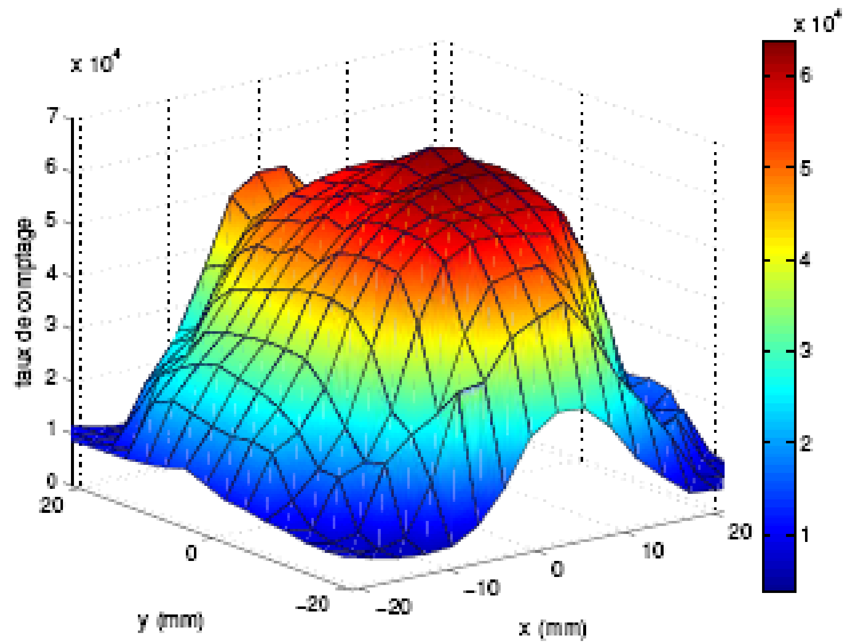
Spectre de photo-électron unique

These de Gwenaëlle Lefeuvre, APC, P7, 2006









Detection of particles and radiation by conversion of dE/dx into light

Typical setup: PM + Scintillator

Light output:

- Inorganic scintillators like NaI : 4 x 10^4 γ / MeV
    - Other crystals 1% to 20% of a NaI

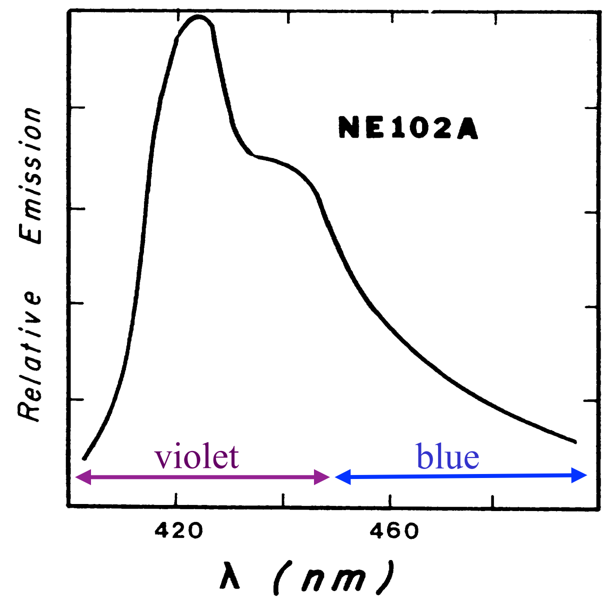
- Organic scintillators produce: O 10^4 γ /MeV (1 γ /100 eV)

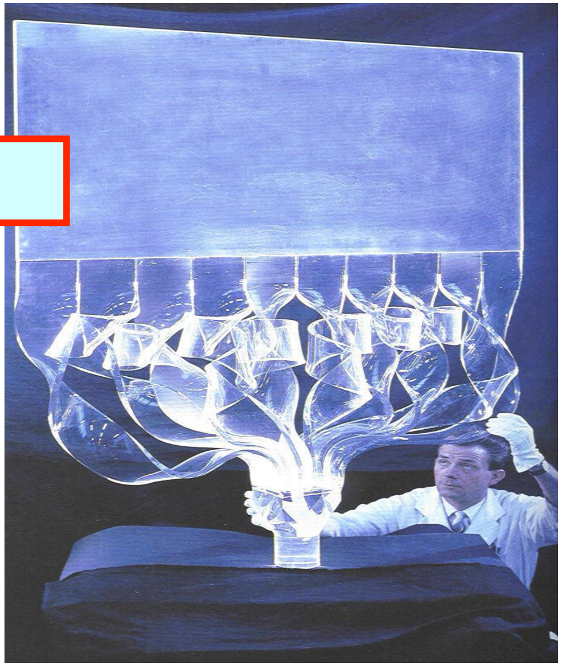
Plastic Scintillators

Type	Light[a] output	λ_{max}[b] (nm)	Attenuation[c] length (cm)	Risetime (ns)	Decay[d] time (ns)	Pulse FWHM (ns)
NE 102A	58–70	423	250	0.9	2.2–2.5	2.7–3.2
NE 104	68	406	120	0.6–0.7	1.7–2.0	2.2–2.5
NE 104B	59	406	120	1	3.0	3
NE 110	60	434	400	1.0	2.9–3.3	4.2
NE 111	40–55	375	8	0.13–0.4	1.3–1.7	1.2–1.6
NE 114	42–50	434	350–400	~1.0	4.0	5.3
Pilot B	60–68	408	125	0.7	1.6–1.9	2.4–2.7
Pilot F	64	425	300	0.9	2.1	3.0–3.3
Pilot U	58–67	391	100–140	0.5	1.4–1.5	1.2–1.9
BC 404	68	408	—	0.7	1.8	2.2
BC 408	64	425	—	0.9	2.1	~2.5
BC 420	64	391	—	0.5	1.5	1.3
ND 100	60	434	400	—	3.3	3.3
ND 120	65	423	250	—	2.4	2.7
ND 160	68	408	125	—	1.8	2.7

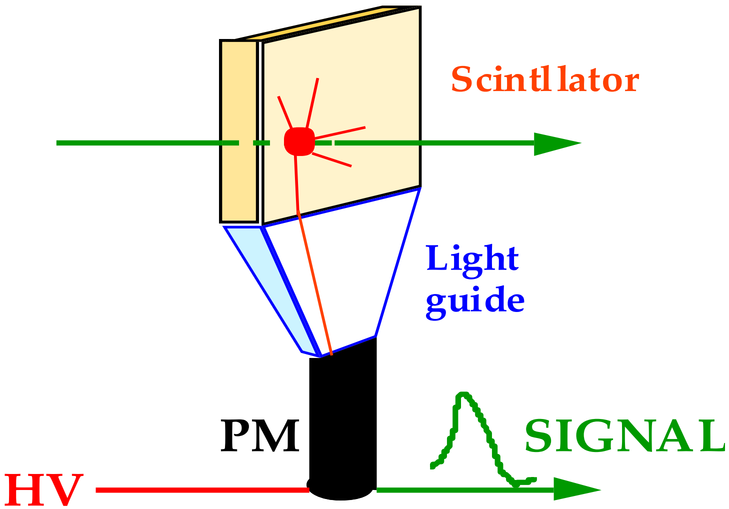
[a] Percentage of anthracene.
[b] Wavelength of maximum emission.
[c] 1/e length.
[d] Main component.

Typical emission spectrum





100 eV/photon



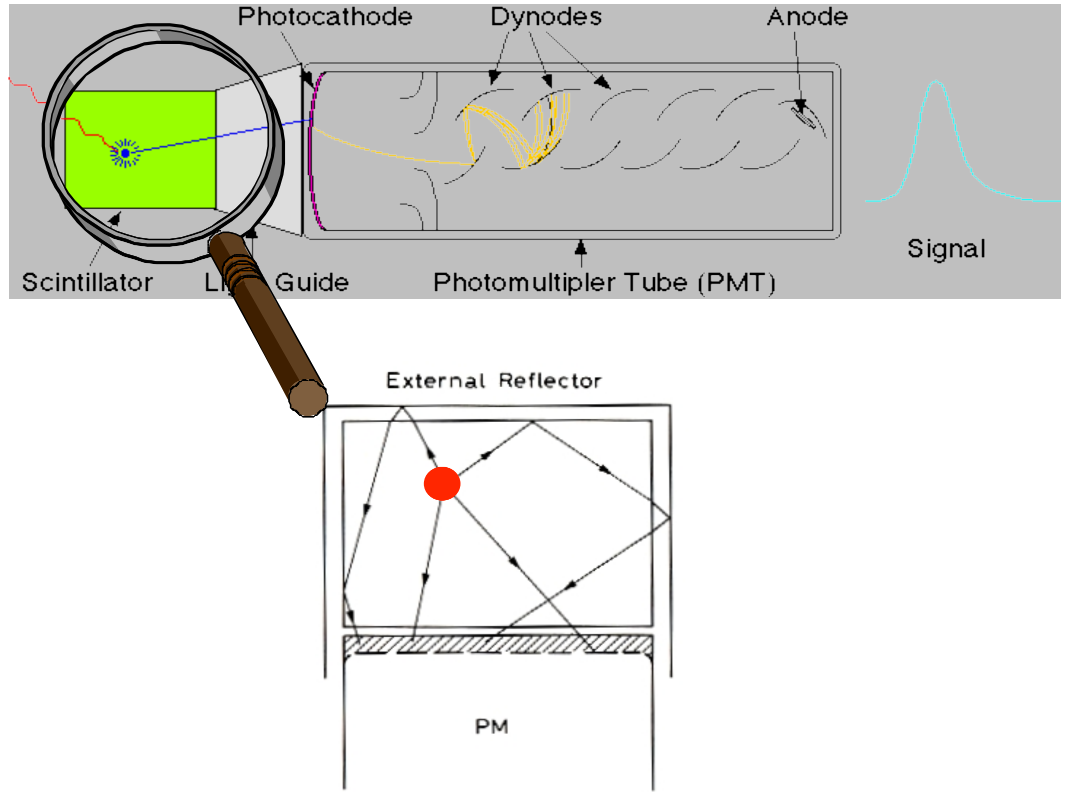
Inorganic Scientillators :

Table 25.2: Properties of several inorganic crystal scintillators.

	NaI(Tl)	BGO	BaF_2	CsI(Tl)	CsI(pure)	$PbWO_4$	CeF_3
Density ($g\ cm^{-3}$):	3.67	7.13	4.89	4.53	4.53	8.28	6.16
Radiation length (cm):	2.59	1.12	2.05	1.85	1.85	0.89	1.68
Molière radius (cm):	4.5	2.4	3.4	3.8	3.8	2.2	2.6
dE/dx (MeV/cm) (per mip):	4.8	9.2	6.6	5.6	5.6	13.0	7.9
Nucl. int. length (cm):	41.4	22.0	29.9	36.5	36.5	22.4	25.9
Decay time (ns):	250	300	0.7^f 620^s	1000	$10, 36^f$ $\sim 1000^s$	5–15	10–30
Peak emission λ (nm):	410	480	220^f 310^s	565	305^f $\sim 480^s$	440–500	310–340
Refractive index:	1.85	2.20	1.56	1.80	1.80	2.16	1.68
Relative light output:*	1.00	0.15	0.05^f 0.20^s	0.40	0.10^f 0.02^s	0.01	0.10
Hygroscopic:	very	no	slightly	somewhat	somewhat	no	no

* For standard photomultiplier tube with a bialkali photocathode. See Ref. 21 for photodiode results.
f = fast component, s = slow component

(Example NaI) : 25 eV / photon



Résolution attendue avec un NaI:

La statistique d 'ionisation et d 'excitation est de type **Poissonnienne**

$$N_{Ionisation} = \frac{E}{w}$$

Avec une variance $\sigma^2 = N_{Ionisation}$ ($N_{ionisation}$ = nombre moyen d 'ionisation)

$$R = 2.35 \frac{\sqrt{N_{Ionisation}}}{N_{Ionisation}} = 2.35\sqrt{\frac{w}{E}} = 2.35\sqrt{\frac{1}{N}}$$

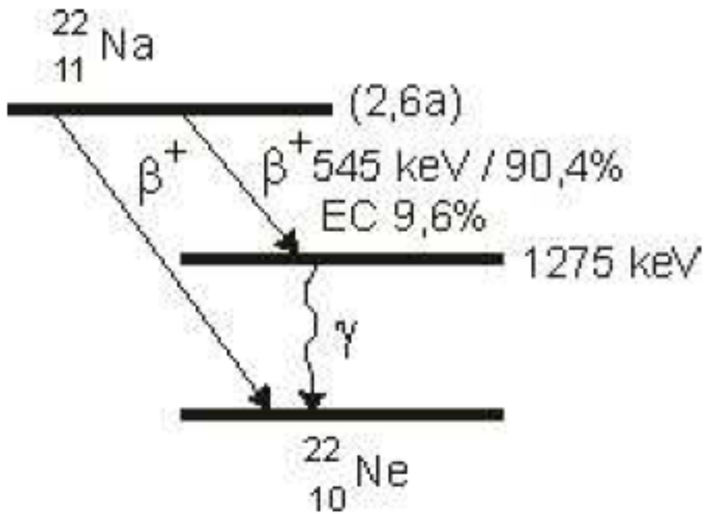
$N_{ionisation}$ = 1photon / 25 eV 1 γ de 511 keV génère 2 x 10^4 photons

Efficacité de collection = 50 %
Efficacté quantique de la photocathode = 20 %

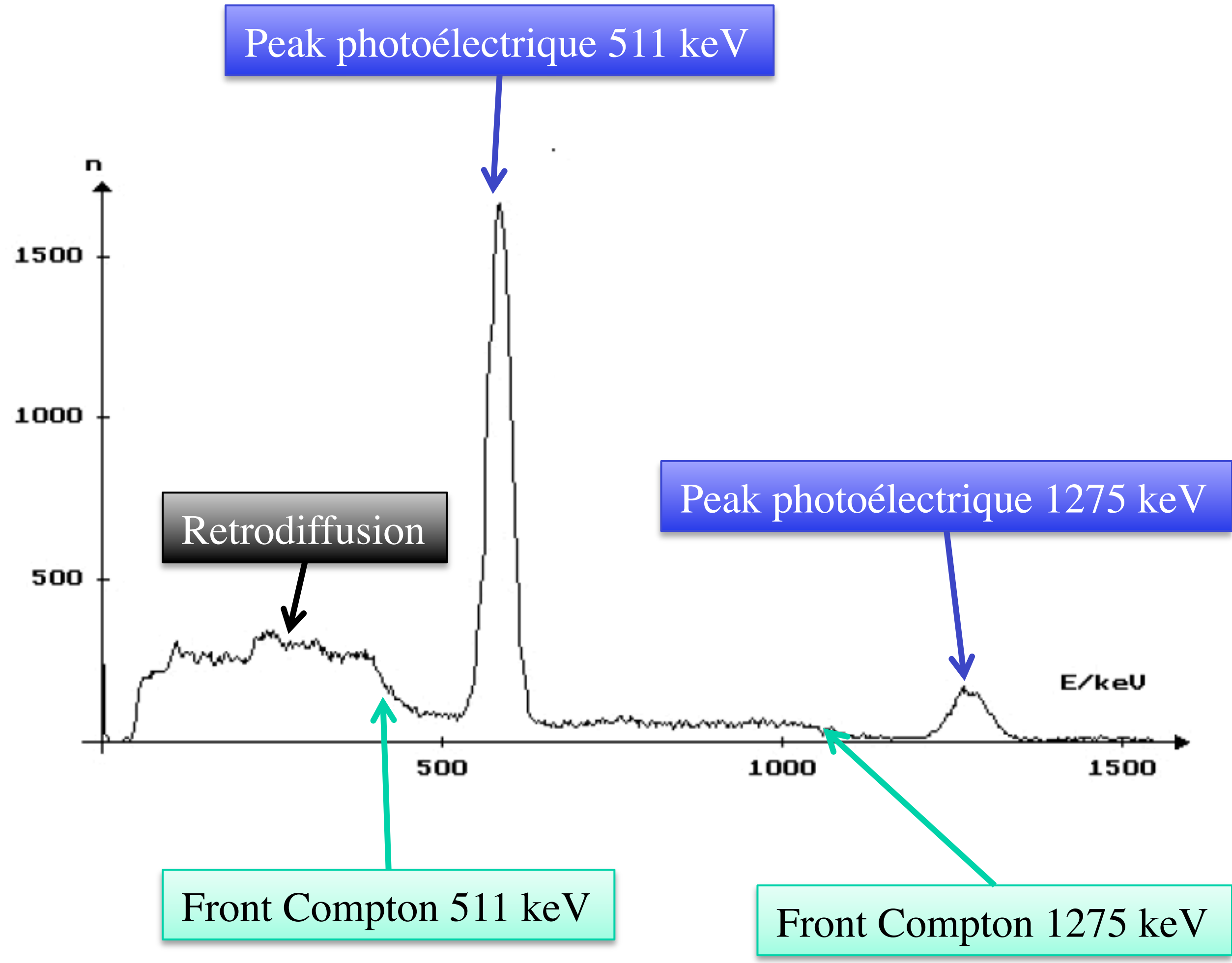
Nombre d'électrons dans le PM = 2 x 10^4 x 0,5 x 0,2 = 2000 photoélectrons

R = 2.35 x sqrt(1/2000) = 5,2 %

^{22} Na:



22 Na 11
(2,6a)
β+
β+ 545 keV / 90,4%
EC 9,6%
1275 keV
γ
22 Ne 10



Peak photoélectrique 511 keV
n
1500
1000
500
Retrodiffusion
Peak photoélectrique 1275 keV
E/keV
500
1000
1500
Front Compton 511 keV
Front Compton 1275 keV

Exemple: PM couplé à un scintillateur plastique:

Quelques paramètres typiques d 'un scintillateur plastique:

Perte d 'énergie	2MeV/cm
Efficacité de scintillation	1 photon/100 eV
Efficacité de collection (nombre de photons arrivés au PM)	0,1
Efficacité quantique du PM	0,25

Quel signal électrique peut-on attendre avec un scintillateur de 1 cm?

Une particule chargée traversant le scintillateur perd 2 MeV, donc crée 2×10^4 photons

$2\times10^4 \times 0{,}1 = 2\times10^3$ photons arrivent au PM qui les transforme en $2\times10^3 \times 0{,}25 =$ 500 électrons

Avec un gain de 10^6: $500 \times 10^6 = 5\times10^8$ électrons $= 8 \times 10^{-11}$ C

Si la charge est collectée en 50 ns —> $I = dq/dt = 8 \times 10^{-11}\ \mathrm{C} / 5 \times 10^{-8}\ \mathrm{s} = 1{,}6 \times 10^{-3}$ A

Ce courant traverse une résistance de 50 Ω —> $V = IR = (50\ \Omega)(1.6\times10^{-3}\mathrm{A}) =$ **80mV**

Visible avec un oscilloscope!

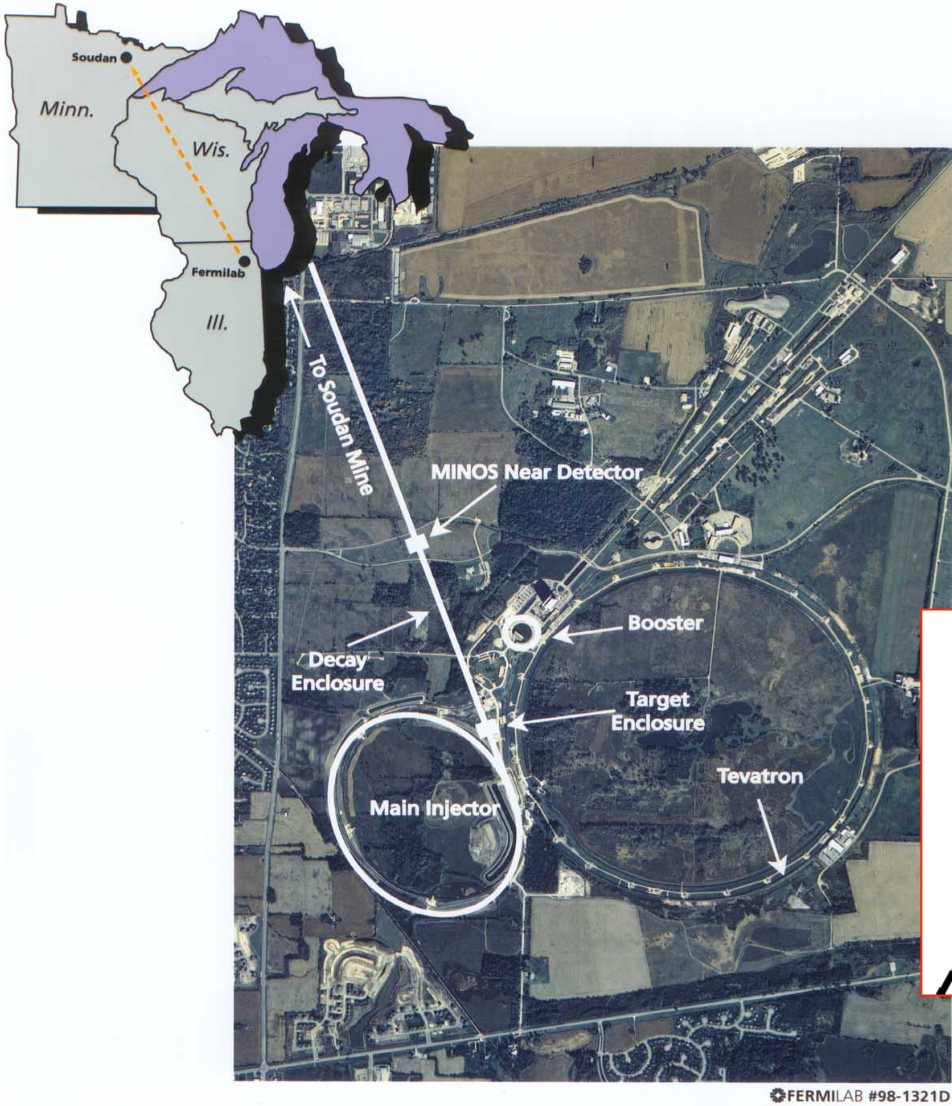
Quelle est l 'efficacité de ce compteur? = Quelle est la probabilité d 'avoir 0 photoélectrons?

Statistique = Poisson:

$$P(r) = \frac{\mu^r e^{-\mu}}{r!} \rightarrow P(O) = \frac{500^0 e^{-500}}{0!} \cong O$$

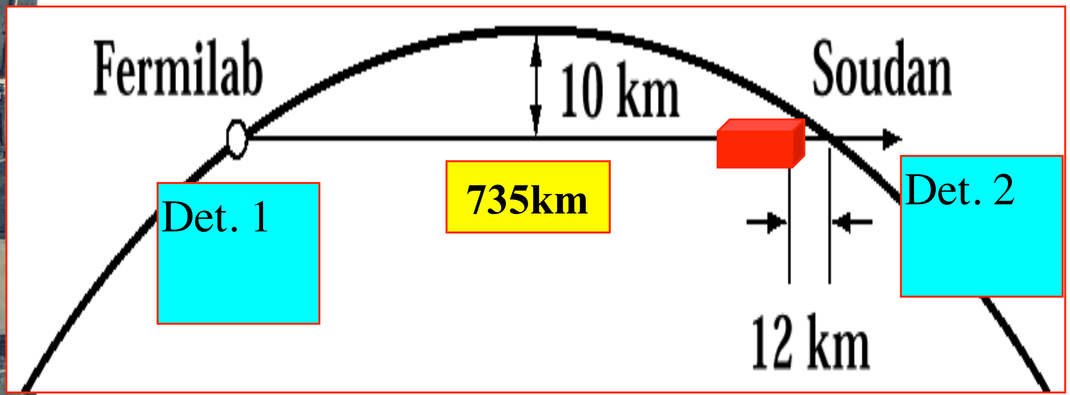
Donc l 'efficacité est de 100%

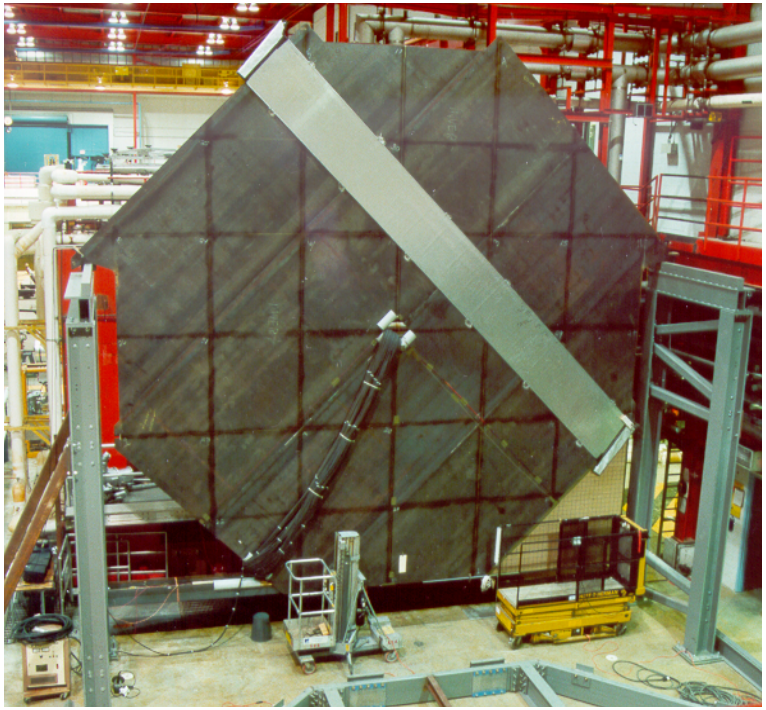
Exemple 1: détecteur MINOS - oscillations du neutrino

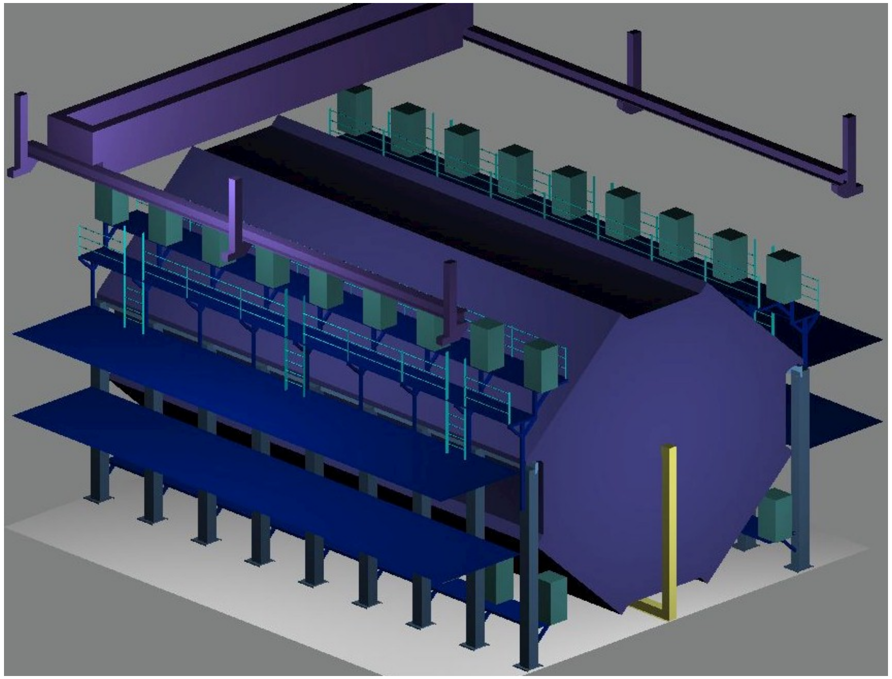


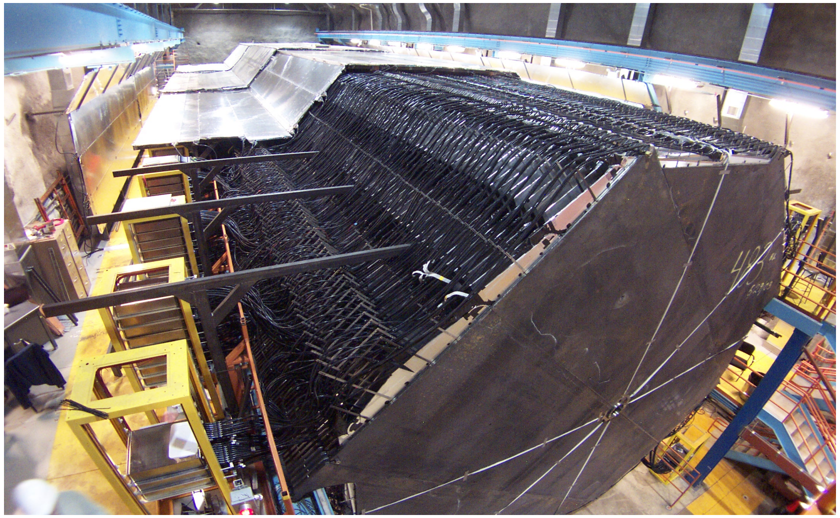
MINOS:Composé de :

- **Un faisceau de neutrino (3 faisceaux!)**
- **Un détecteur proche (980 t @ 1 km)**
- **Un détecteur lointain (5,4 kt @ 730 km)**









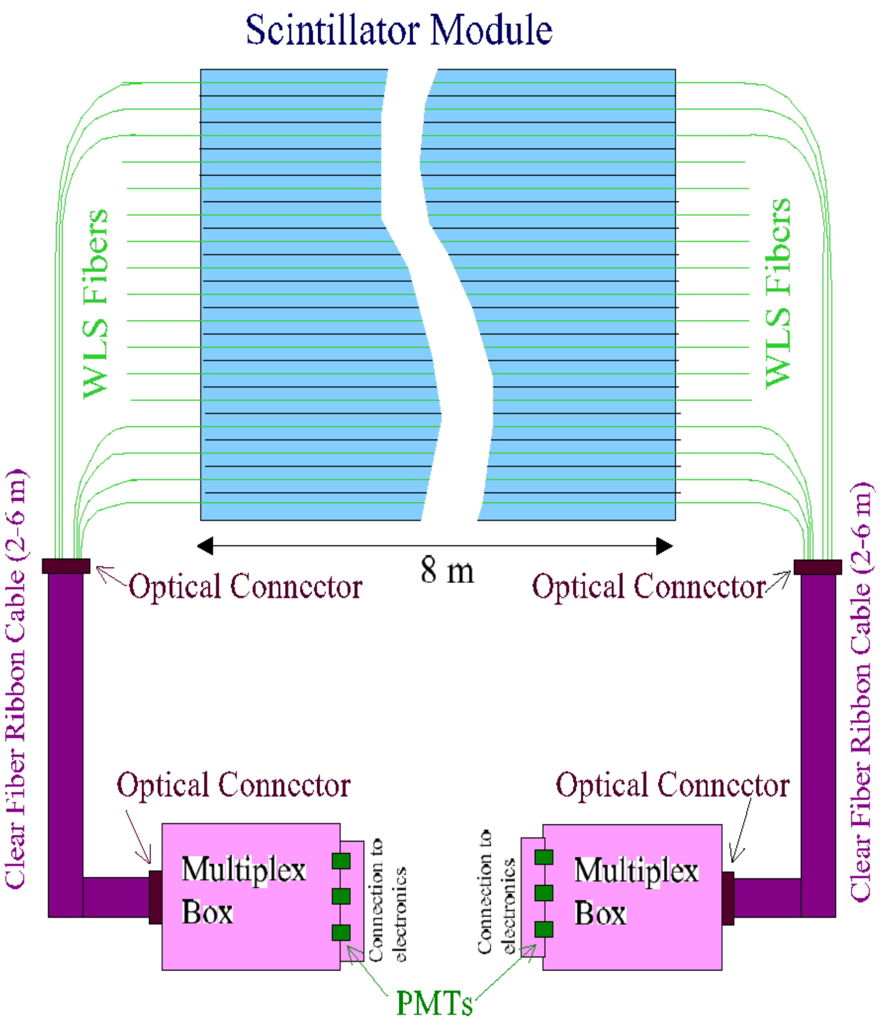
Constitué de :

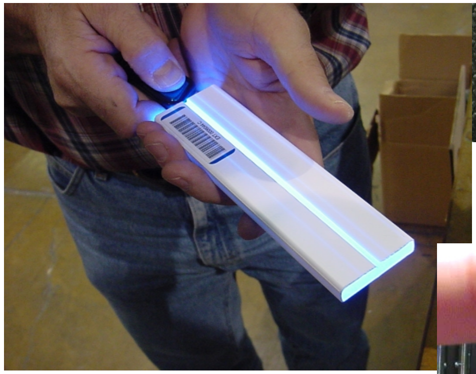
485 plaques d'acier octogonales (5,14 kt)

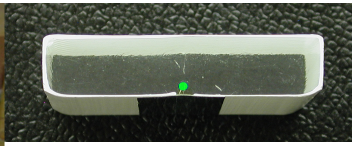
484 plaques de scintillateur octogonales (0,26 kt)

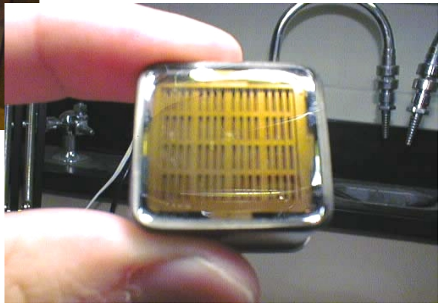
- 92,928 strips (4.1 x 1.0 cm), 1452 M16s
- 722 km de fibre (WLS fiber)
- 794 km fibre (clear fiber)

Un champ magnétique de 1,5 Tesla!

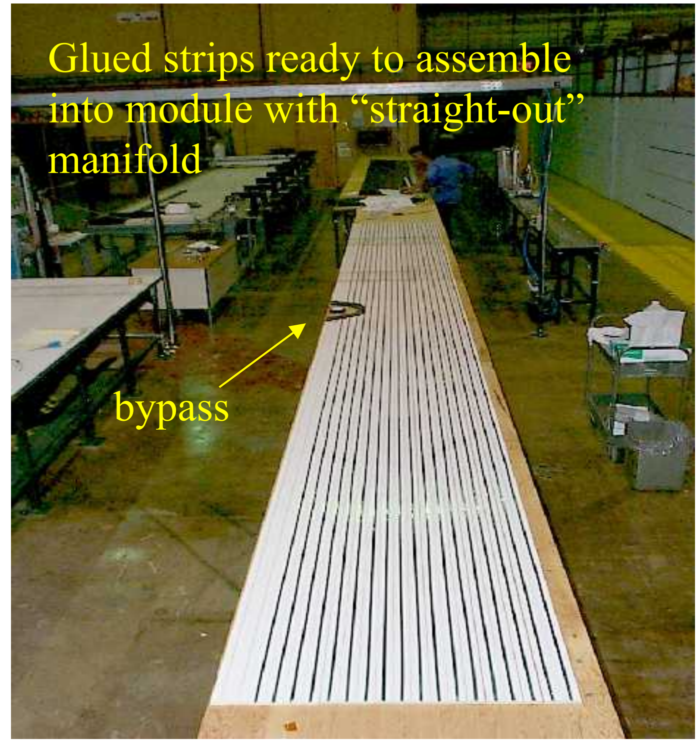

Scintillator Module
WLS Fibers
WLS Fibers
8 m
Optical Connector
Optical Connector
Clear Fiber Ribbon Cable (2-6 m)
Clear Fiber Ribbon Cable (2-6 m)
Optical Connector
Optical Connector
Multiplex Box
Multiplex Box
Connection to electronics
Connection to electronics
PMTs

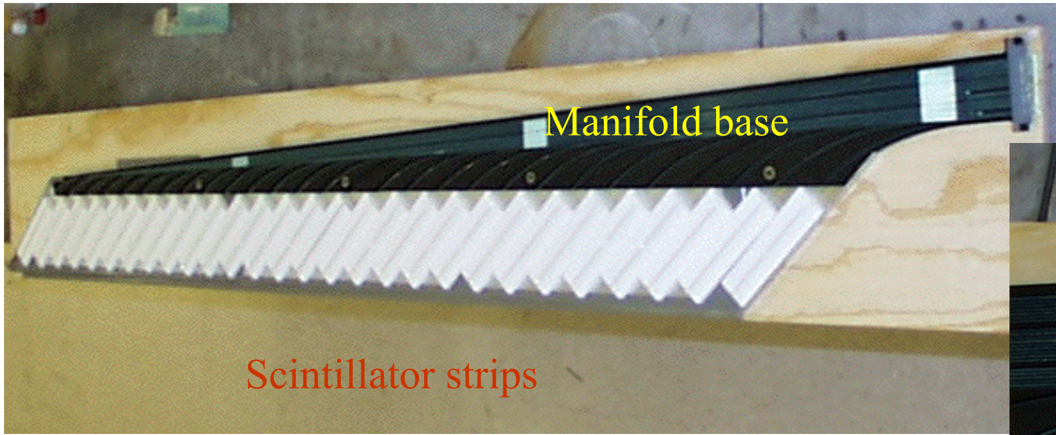


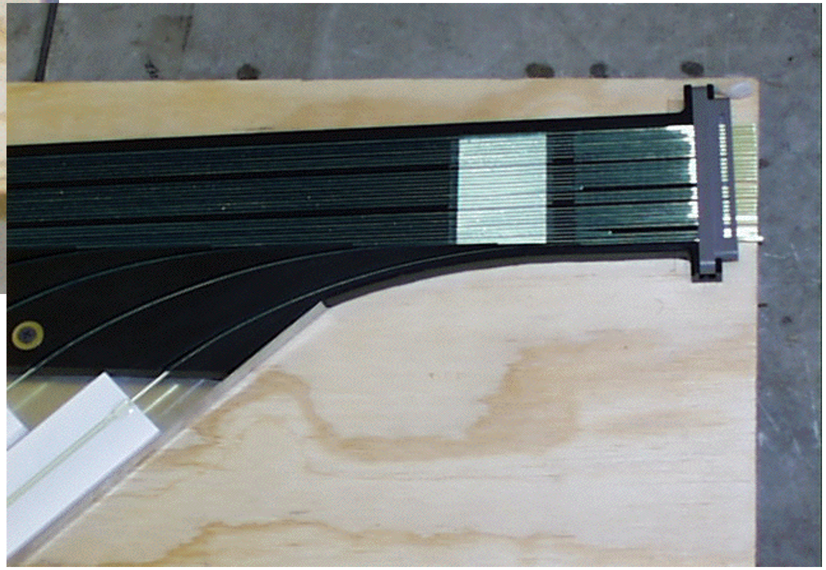


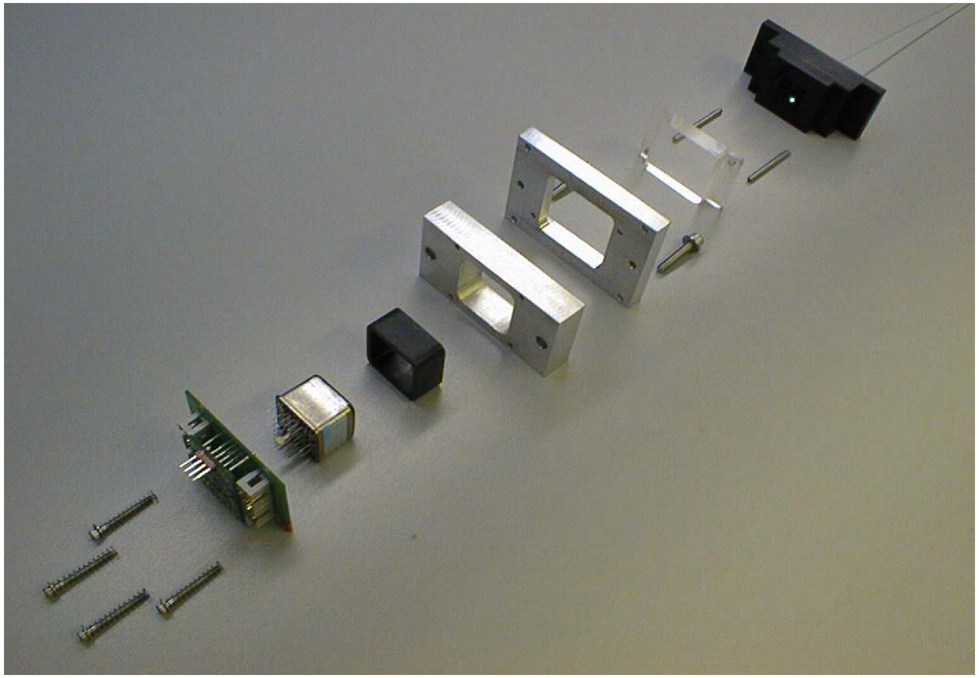


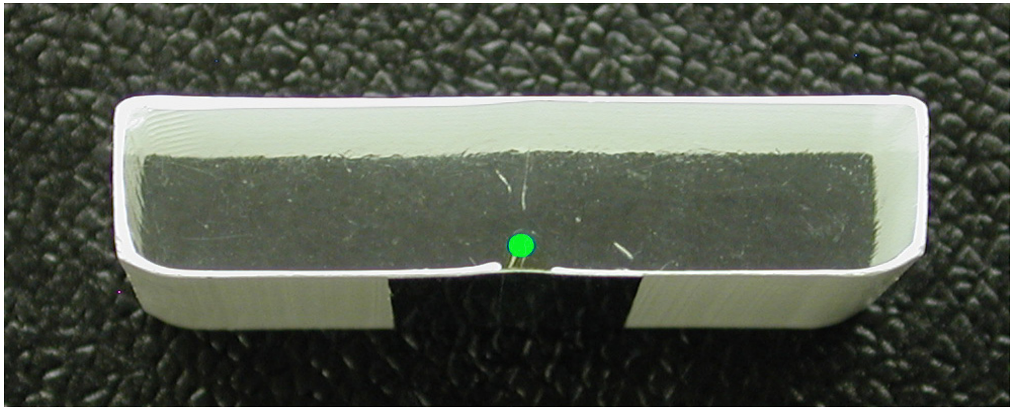

Light Case
Manifold base with fiber grooves
Variable width seal
Light injection cover
Formed Al cover
Optical
connector

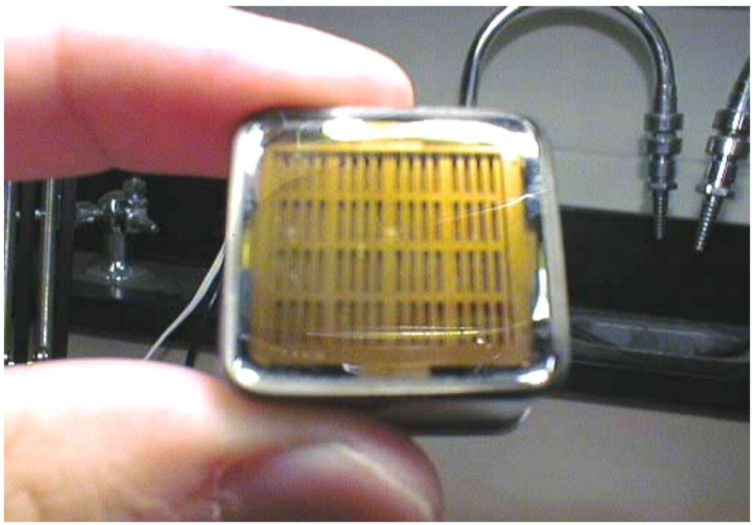

Glued strips ready to assemble
into module with "straight-out"
manifold
bypass

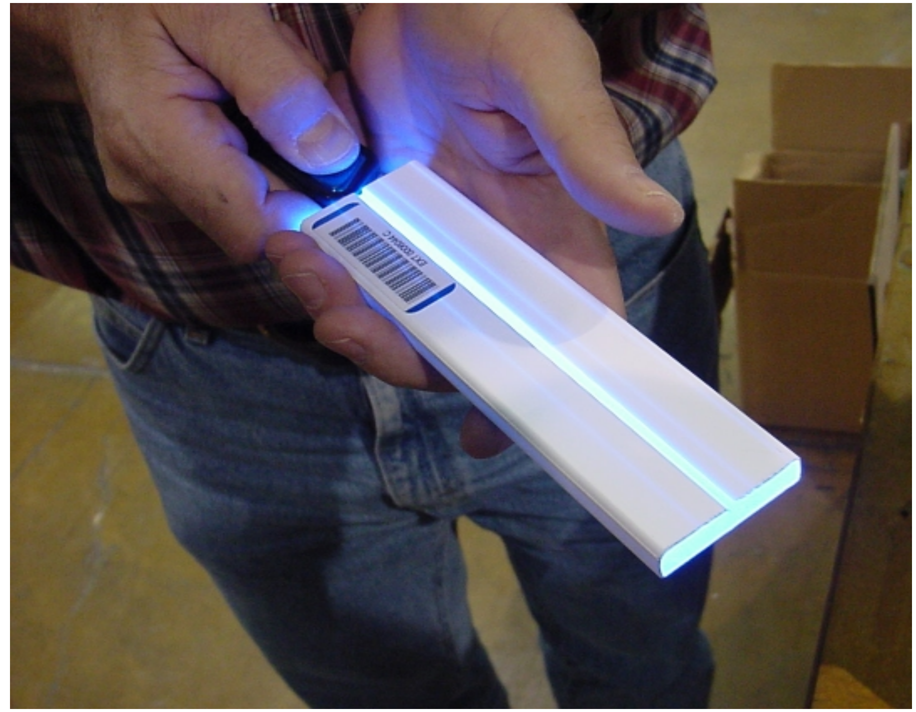

Manifold base
Scintillator strips



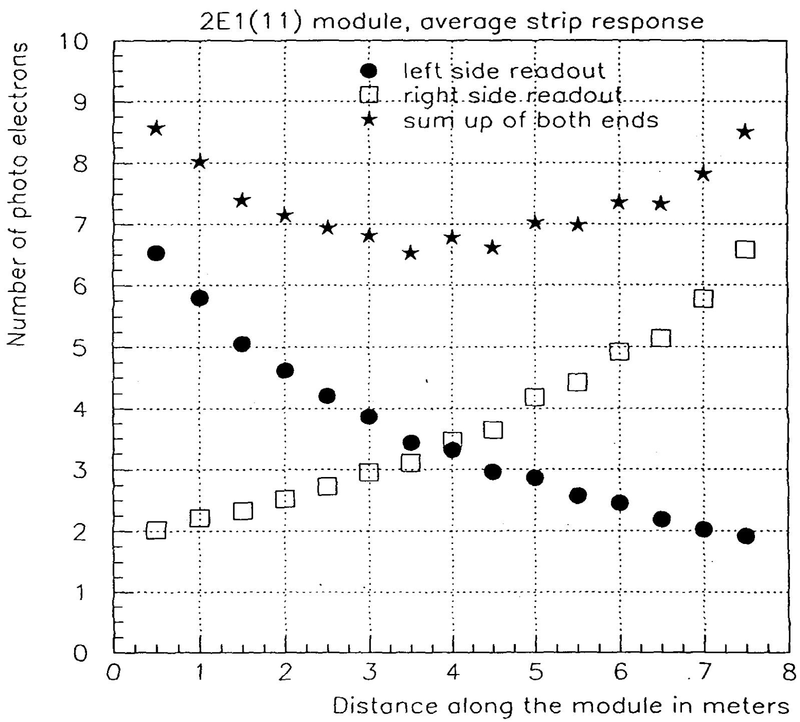




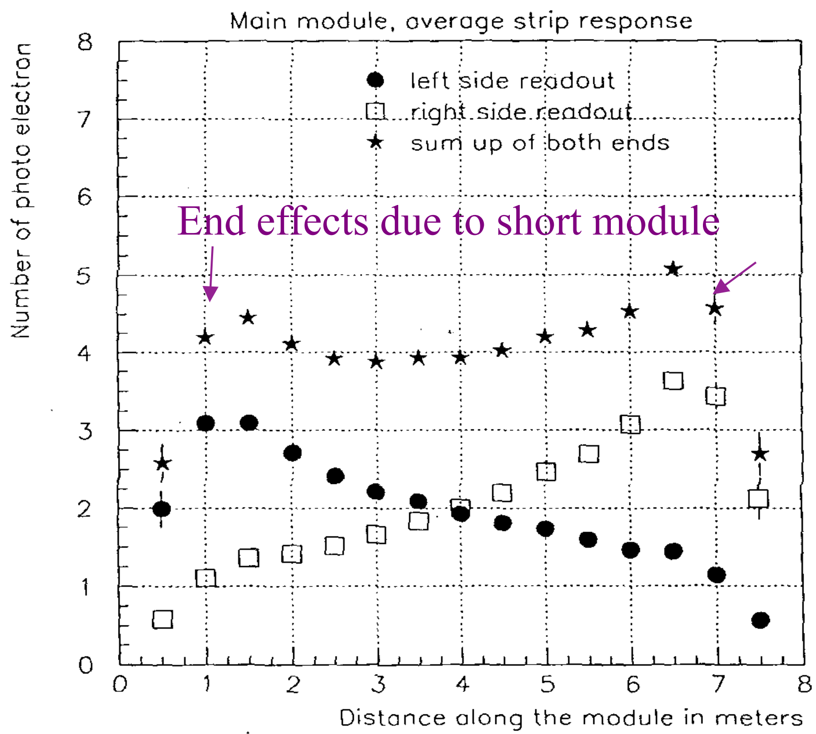




Résultats de test: nombre des photo-électrons par particule d 'ionisation minimale

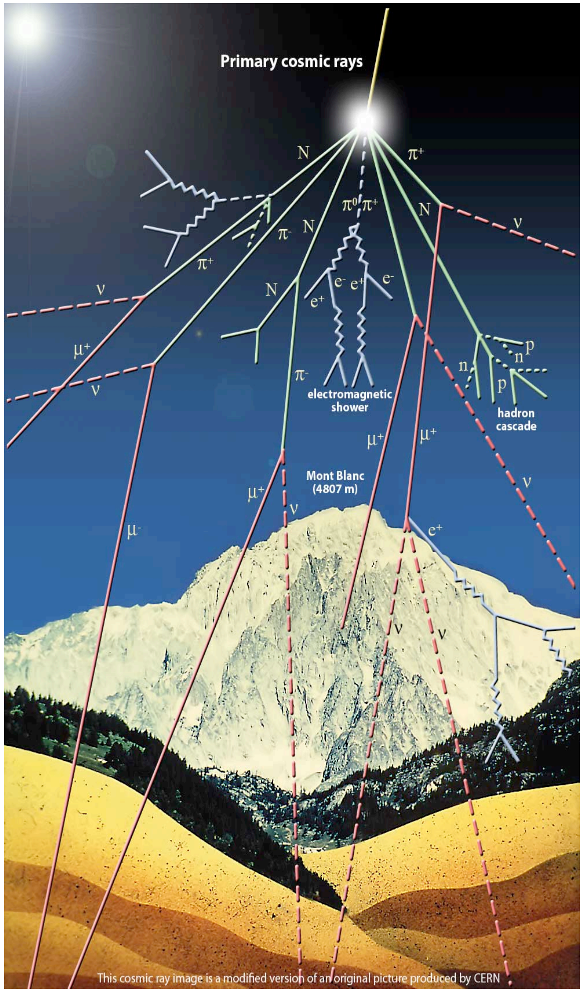


Meilleur module



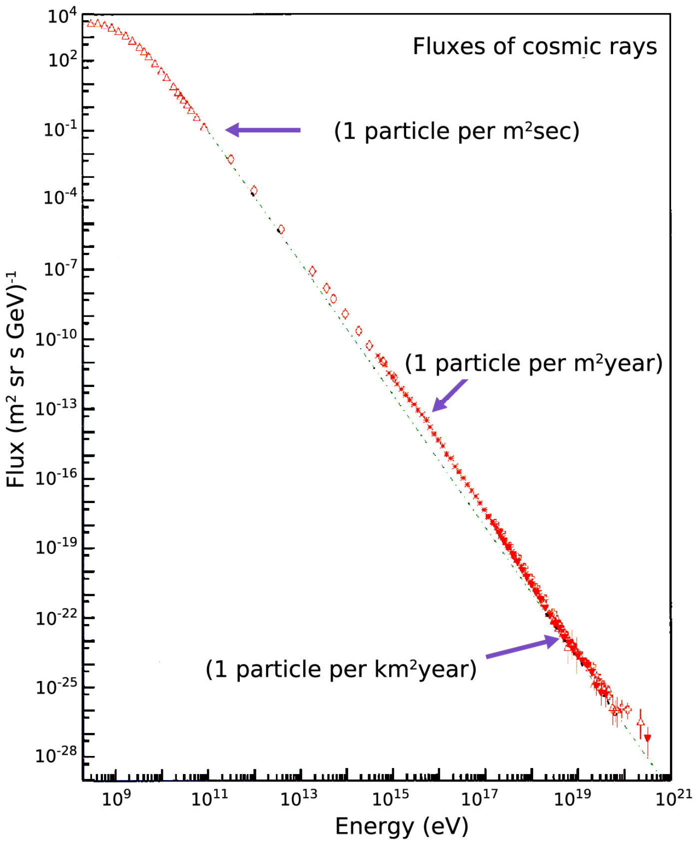
Mauvaise module

Use of atmospheric muons – a cheep and easy way for calibration…

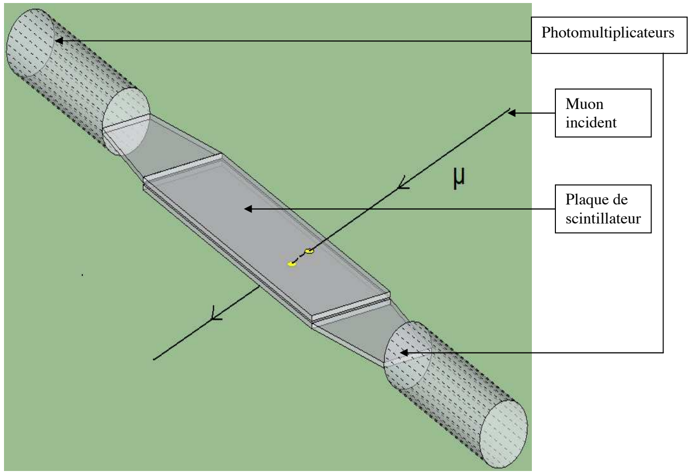


$< E\mu_{atm} > \approx 4$ GeV

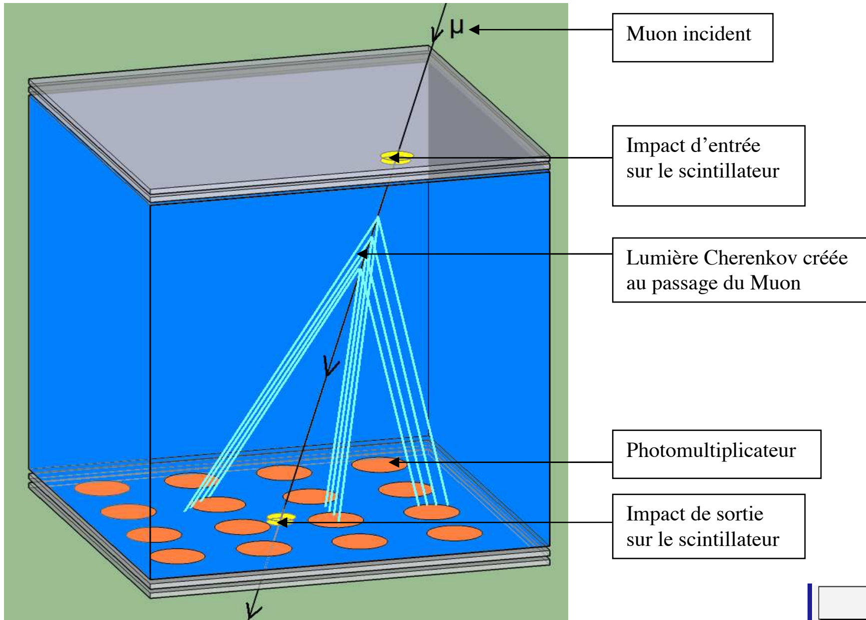
Flux at ground $\approx 130\ m^{-2}\ s^{-1}$

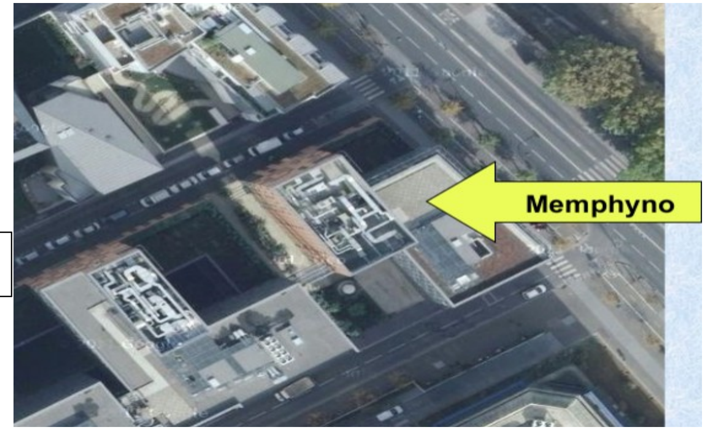


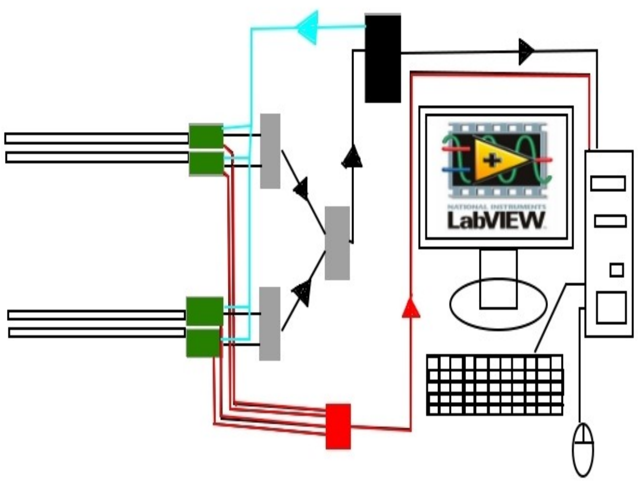
μ - hodoscope

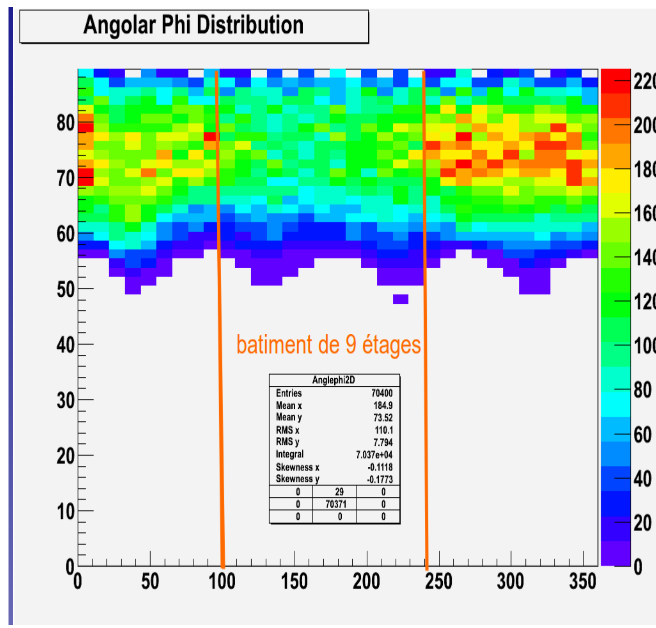


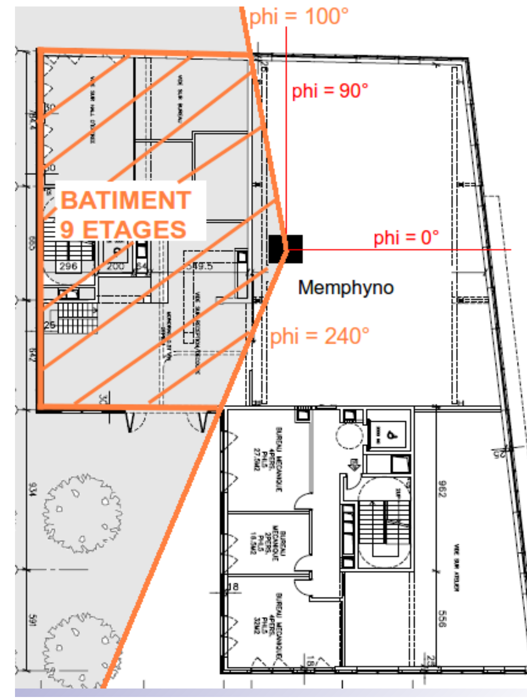
Use of atmospheric muons – a cheep and easy way for calibration…











MIP's (Minimum Ionizing Particles)

Very useful tool for calibration of experiments
And
Discussions…

Exemple 2: Application en médecine

Les principes de la tomographie à émission de positrons (TEP)

Source: M-L Gallin-Martel, ISN, IN2P3

Etape 1 : Production du traceur

- Isotopes standards émetteurs β^+ ⇒

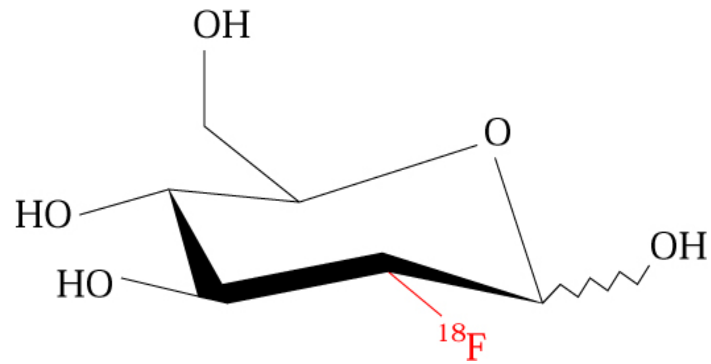
	^{18}F	^{15}O	^{11}C	^{13}N
T :	114 min.	2 min.	20 min.	10 min.

Etape 2 : Synthèse du radio traceur

Marquage d 'un composé biologique

EX : Fluorodésoxyglucose marqué ^{18}F ⇒ FDG
90 % des radio pharmaceutiques utilisés en TEP

Radio Synthèse : Introduction du ^{18}F sur une liaison carbone



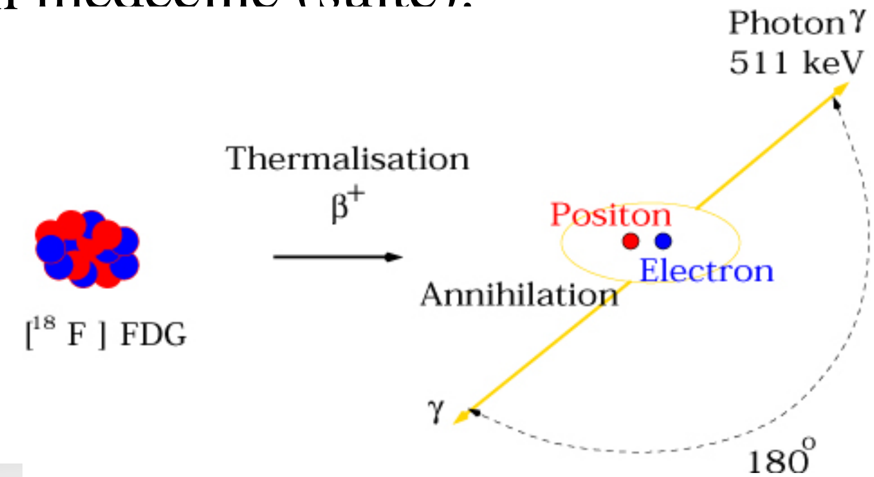
Exemple: Application en médecine (suite):

Etape 3 : Processus physiques

1♦ Désintégration β^+ du traceur

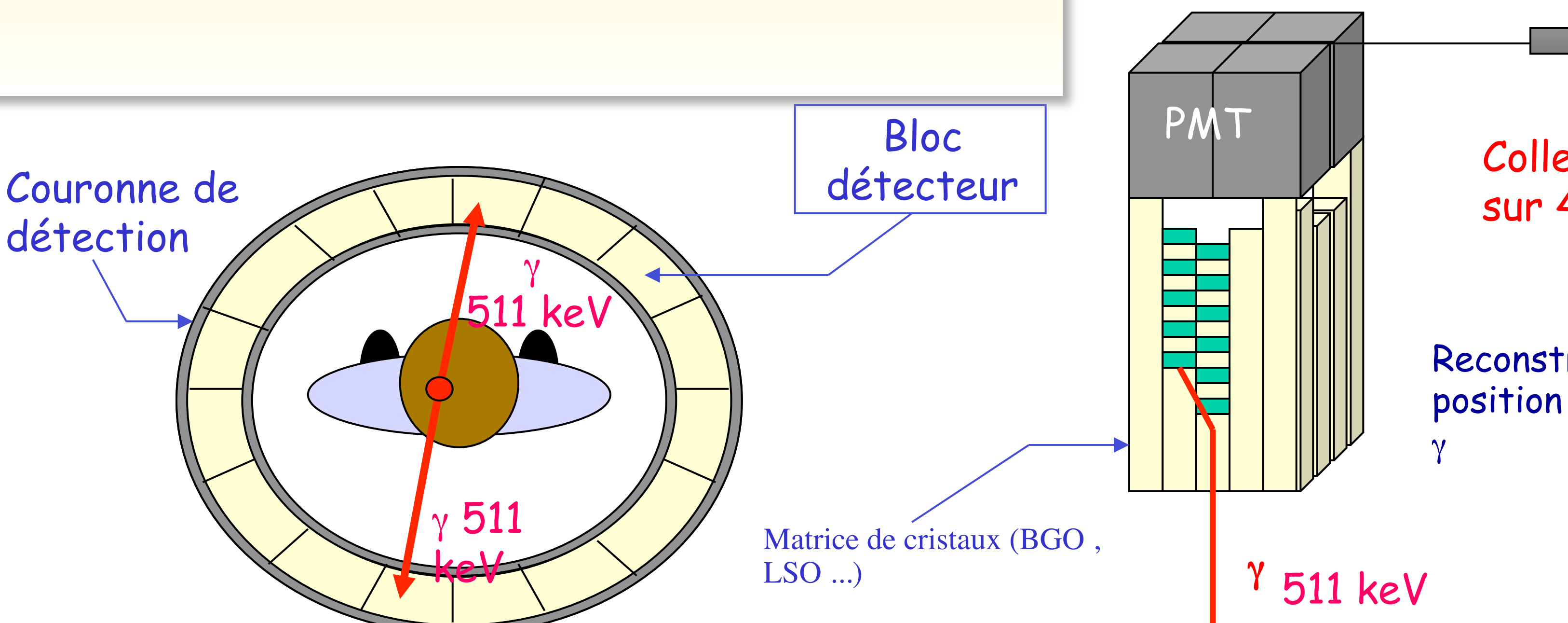
2♦ Thermalisation du β^+ dans les tissus

3♦ Annihilation : $e^+e^- \rightarrow \gamma\ \gamma$



Etape 4 : Détection et acquisition du signal

♦ Détection des γ en coïncidence ♦ Collimation électronique



Conclusions:

- The basics of interactions of particles and radiation with matter are reviewed.
- One should get the order of magnitude of the expected signal from a « back of the envelope" calculation.

- For this conference examples on light detection are chosen.
- Many application in LEP, HEP, medical imaging, environmental surveillance and many more.

Thanks to all authors of illustrations used in the presentation and found on the web!

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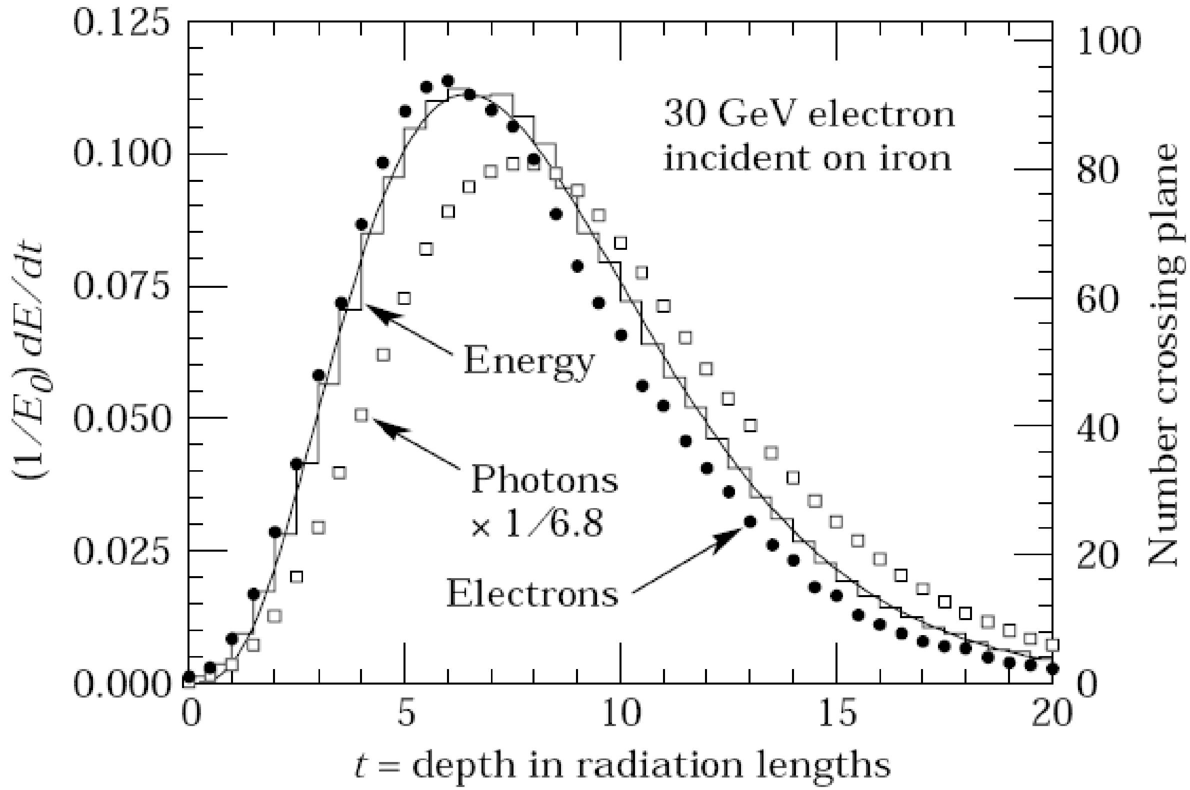

0.125
0.100
0.075
0.050
0.025
0.000
(1/E0) dE/dt
100
80
60
40
20
0
Number crossing plane
0
5
10
15
20
t = depth in radiation lengths
30 GeV electron
incident on iron
Energy
Photons
× 1/6.8
Electrons

Bremsstrahlung ⟷ **Cherenkov radiation**

Bremsstrahlung	Cherenkov radiation
Origin: Acceleration of a particle in the field of the nucleus	Origin: Polarization of the material after passage of the particle
$$Intensity \propto \frac{z^2 Z^2}{M}$$	Intensity is independent of the particle mass
$$\theta \propto \frac{m_0 c^2}{E_0}$$	$$\cos\theta = \frac{1}{\beta n}$$

Compton scattering:

1929: Klein-Nishima formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2[1+\gamma(1-\cos\theta]^2}\left(1+\cos^2\theta+\frac{\gamma^2(1-\cos\theta)^2}{1+\gamma(1-\cos\theta)}\right)$$

with $\gamma = h\nu / m_e c^2$

★ High energy limit ($\gamma >> 1$) all photons are foreward scattered ($\theta = 0$)

★ **Thomson scattering** (classical limit of scattering of photons by free electrons) – Klein –Nishime reduces to

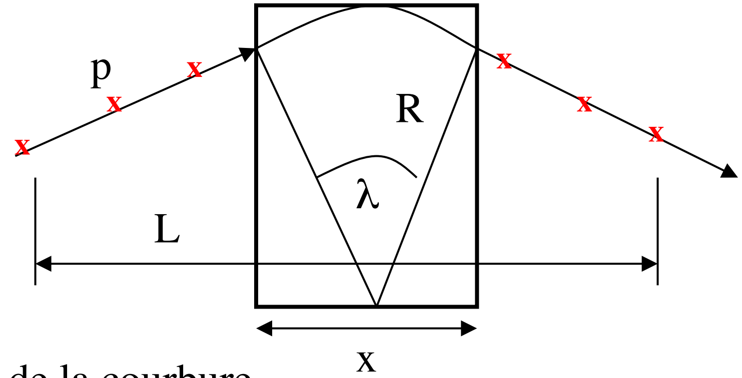
$$\sigma = \frac{8\pi}{3} r_e^2$$ Thomson cross section

Rayleigh scattering = scattering of photons by atoms as a whole (all electrons contribute) = coherent scattering

Measurement of particle momentum in a magnetic field:

$P\cos\lambda = 0.3\,z\,B\,R$

(R [m], rayon de courbure et
B [Tesla], champ magnétique)



La distribution des mesures de la courbure **k = 1/R** est ≈ gaussienne

$$(\delta k)^2 = (\delta k_{res})^2 + (\delta k_{ms})^2$$

δk = erreur de la courbure
δk_{res} = erreur de la résolution
δk_{ms} = erreur de la diffusion multiple

Mesure le long de la trace de N>10 points avec une erreur σ(x) par point :

$$\delta k_{res} = \frac{\sigma(x)}{L^2}\sqrt{\frac{720}{N+4}}$$

L = projection de la longueur
σ(x) = erreur de la mesure de chaque point de la trace

La résolution en impulsion sera affectée par la diffusion multiple

$$\delta k_{ms} \approx \frac{(0.016)(GeV/c)z}{Lp\beta\cos^2\lambda}\sqrt{\frac{L}{X_0}}$$

Et aussi: $\delta k_{ms} \approx 8 s^{rms}_{plane} / L^2$

Résolution pour l 'impulsion ⇒

$$\left|\frac{\Delta p}{p}\right| = \frac{p}{0.3B}\,\delta k$$

Mesure de l 'impulsion en champ magnétique

Exemple: expérience CHORUS (CERN)

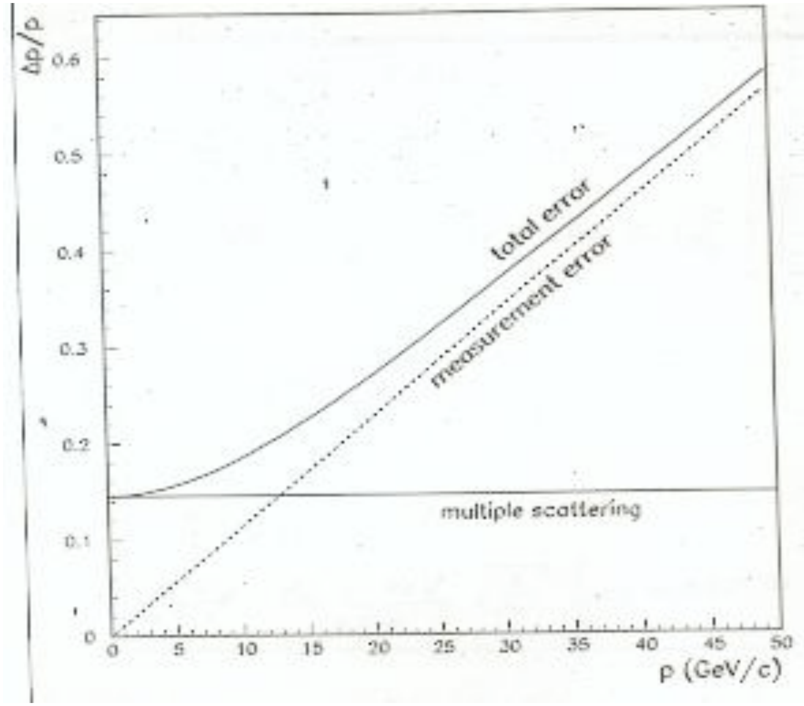
$\sigma(x)$ = 1 mm = 10^{-3} m, L = 1,3 m, x = 0,5 m, B = 1,65 T, 4 points de mesure

$$\delta k_{res} = \frac{\sigma(x)}{L^2}\sqrt{\frac{720}{N+4}} = \frac{10^{-3}}{1.69}\sqrt{\frac{720}{8}} = 5.61\times 10^{-2}$$

$$\left|\frac{\Delta p}{p}\right|_{res} = \delta k_{res}\times\frac{p}{0.3\times 1.65} = 1.13\times 10^{-2}\times p$$

$$\delta k_{ms} = \frac{1}{L^2}8\frac{1}{4\sqrt{3}}x\theta_0\left(\frac{1}{p}\right) = \frac{1.154}{1.69}\times 0.5\times 0.2112\left(\frac{1}{p}\right) = 0.0721\left(\frac{1}{p}\right)$$

$$\left|\frac{\Delta p}{p}\right|_{ms} = \delta k_{ms}\frac{p}{0.3\times 1.65} = 0.1456$$



Erreur totale:

$$\left|\frac{\Delta p}{p}\right| = \sqrt{\left|\frac{\Delta p}{p}\right|_{res}^2 + \left|\frac{\Delta p}{p}\right|_{ms}^2} = \sqrt{1.277\times 10^{-4}p^2 + 0.0212}$$